11 Rolling-Contact Bearings

Chapter Outline

The terms *rolling-contact bearing, antifriction bearing,* and *rolling bearing* are all used to describe that class of bearing in which the main load is transferred through elements in rolling contact rather than in sliding contact. In a rolling bearing the starting friction is about twice the running friction, but still it is negligible in comparison with the starting friction of a sleeve bearing. Load, speed, and the operating viscosity of the lubricant do affect the frictional characteristics of a rolling bearing. It is probably a mistake to describe a rolling bearing as "antifriction," but the term is used generally throughout the industry.

From the mechanical designer's standpoint, the study of antifriction bearings differs in several respects when compared with the study of other topics because the bearings they specify have already been designed. The specialist in antifriction-bearing design is confronted with the problem of designing a group of elements that compose a rolling bearing: these elements must be designed to fit into a space whose dimensions are specified; they must be designed to receive a load having certain characteristics; and finally, these elements must be designed to have a satisfactory life when operated under the specified conditions. Bearing specialists must therefore consider such matters as fatigue loading, friction, heat, corrosion resistance, kinematic problems, material properties, lubrication, machining tolerances, assembly, use, and cost. From a consideration of all these factors, bearing specialists arrive at a compromise that, in their judgment, is a good solution to the problem as stated.

We begin with an overview of bearing types; then we note that bearing life cannot be described in deterministic form. We introduce the invariant, the statistical distribution of bearing life, which is described by the Weibull distribution. There are some useful deterministic equations addressing load versus life at constant reliability, and the catalog rating as rating life is introduced. The load-life-reliability relationship combines statistical and deterministic relationships, which gives the designer a way to move from the desired load and life to the catalog rating in *one* equation.

Ball bearings also resist thrust, and a unit of thrust does different damage per revolution than a unit of radial load, so we must find the equivalent pure radial load that does the same damage as the existing radial and thrust loads. Next, variable loading, stepwise and continuous, is approached, and the equivalent pure radial load doing the same damage is quantified. Oscillatory loading is mentioned.

With this preparation we have the tools to consider the selection of ball and cylindrical roller bearings. The question of misalignment is quantitatively approached.

Tapered roller bearings have some complications, and our experience so far contributes to understanding them.

Having the tools to find the proper catalog ratings, we make decisions (selections), we perform a design assessment, and the bearing reliability is quantified. Lubrication and mounting conclude our introduction. Vendors' manuals should be consulted for specific details relating to bearings of their manufacture.

11–1 Bearing Types

Bearings are manufactured to take pure radial loads, pure thrust loads, or a combination of the two kinds of loads. The nomenclature of a ball bearing is illustrated in Fig. 11–1, which also shows the four essential parts of a bearing. These are the outer ring, the inner ring, the balls or rolling elements, and the separator. In low-priced bearings, the separator is sometimes omitted, but it has the important function of separating the elements so that rubbing contact will not occur.

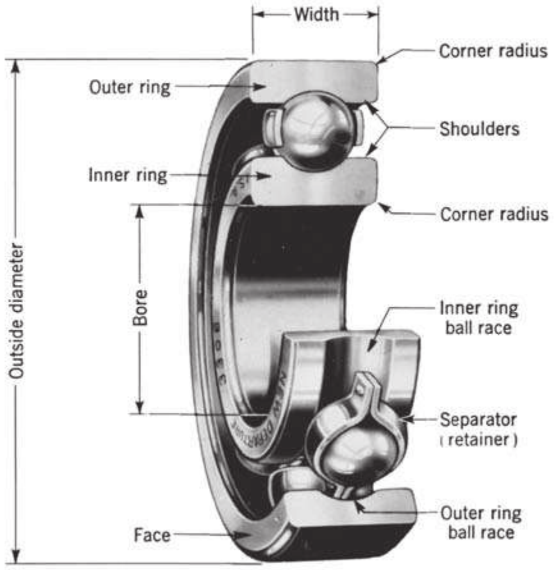


Figure 11–1

Nomenclature of a ball bearing. *(General Motors Corp. Used with permission, GM Media Archives.)*

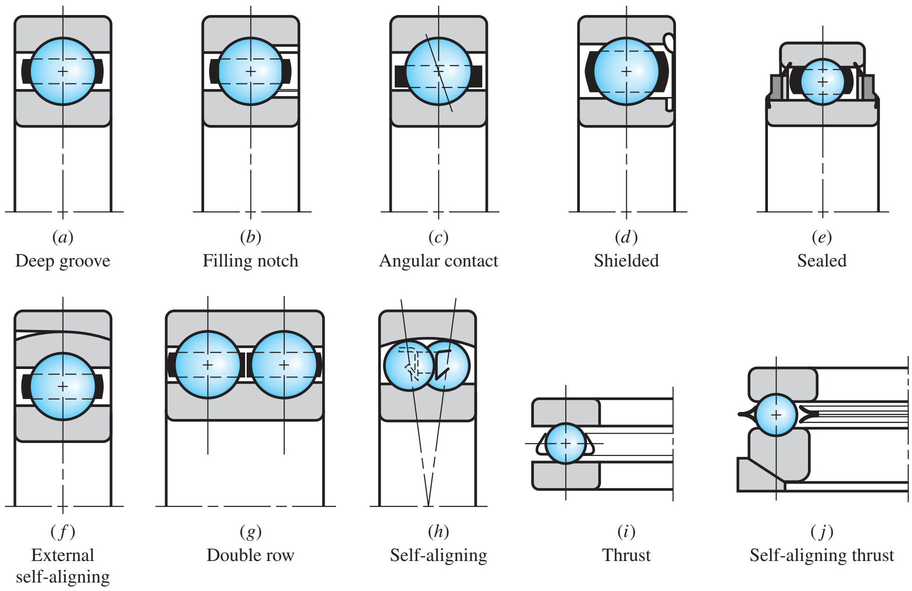


Figure 11–2

Various types of ball bearings.

In this section we include a selection from the many types of standardized bearings that are manufactured. Most bearing manufacturers provide engineering manuals and brochures containing lavish descriptions of the various types available. In the small space available here, only a meager outline of some of the most common types can be given. So you should include a survey of bearing manufacturers' literature in your studies of this section.

Some of the various types of standardized bearings that are manufactured are shown in Fig. 11–2. The single-row deep-groove bearing will take radial load as well as some thrust load. The balls are inserted into the grooves by moving the inner ring to an eccentric position. The balls are separated after loading, and the separator is then inserted. The use of a filling notch (Fig. 11–2*b*) in the inner and outer rings enables a greater number of balls to be inserted, thus increasing the load capacity. The thrust capacity is decreased, however, because of the bumping of the balls against the edge of the notch when thrust loads are present. The angular-contact bearing (Fig. 11–2*c*) provides a greater thrust capacity.

All these bearings may be obtained with shields on one or both sides. The shields are not a complete closure but do offer a measure of protection against dirt. A variety of bearings are manufactured with seals on one or both sides. When the seals are on both sides, the bearings are lubricated at the factory. Although a sealed bearing is supposed to be lubricated for life, a method of relubrication is sometimes provided.

Single-row bearings will withstand a small amount of shaft misalignment of deflection, but where this is severe, self-aligning bearings may be used. Double-row bearings are made in a variety of types and sizes to carry heavier radial and thrust loads. Sometimes two single-row bearings are used together for the same reason, although a double-row bearing will generally require fewer parts and occupy less space. The one-way ball thrust bearings (Fig. 11–2*i*) are made in many types and sizes.

Some of the large variety of standard roller bearings available are illustrated in Fig. 11–3. Straight roller bearings (Fig. 11–3*a*) will carry a greater radial load than ball bearings of the same size because of the greater contact area. However, they have the disadvantage of requiring almost perfect geometry of the raceways and rollers. A slight misalignment will cause the rollers to skew and get out of line. For this reason, the retainer must be heavy. Straight roller bearings will not, of course, take thrust loads.

Helical rollers are made by winding rectangular material into rollers, after which they are hardened and ground. Because of the inherent flexibility, they will take considerable misalignment. If necessary, the shaft and housing can be used for raceways instead of separate inner and outer races. This is especially important if radial space is limited.

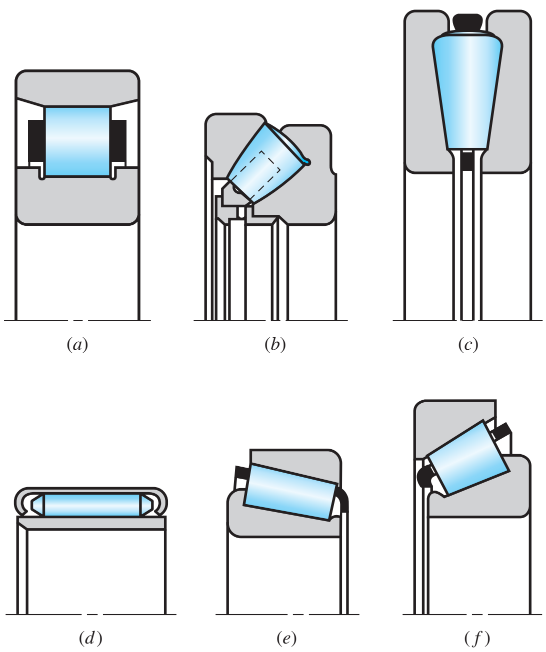


Figure 11–3

Types of roller bearings: (*a*) straight roller; (*b*) spherical roller, thrust; (*c*) tapered roller, thrust; (*d*) needle; (*e*) tapered roller; (*f*) steep-angle tapered roller. *(Courtesy of The Timken Company.)*

The spherical-roller thrust bearing (Fig. 11–3*b*) is useful where heavy loads and misalignment occur. The spherical elements have the advantage of increasing their contact area as the load is increased.

Needle bearings (Fig. 11–3*d*) are very useful where radial space is limited. They have a high load capacity when separators are used, but may be obtained without separators. They are furnished both with and without races.

Tapered roller bearings (Fig. 11–3*e*, *f*) combine the advantages of ball and straight roller bearings, since they can take either radial or thrust loads or any combination of the two, and in addition, they have the high load-carrying capacity of straight roller bearings. The tapered roller bearing is designed so that all elements in the roller surface and the raceways intersect at a common point on the bearing axis.

The bearings described here represent only a small portion of the many available for selection. Many special-purpose bearings are manufactured, and bearings are also made for particular classes of machinery. Typical of these are:

- Instrument bearings, which are high-precision and are available in stainless steel and high-temperature materials
- Nonprecision bearings, usually made with no separator and sometimes having split or stamped sheet-metal races
- Ball bushings, which permit either rotation or sliding motion or both
- Bearings with flexible rollers

11–2 Bearing Life

When the ball or roller of rolling-contact bearings rolls, contact stresses occur on the inner ring, the rolling element, and on the outer ring. Because the curvature of the contacting elements in the axial direction is different from that in the radial direction, the equations for these stresses are more involved than in the Hertz equations presented in Chap. 3. If a bearing is clean and properly lubricated, is mounted and sealed against the entrance of dust and dirt, is maintained in this condition, and is operated at reasonable temperatures, then metal fatigue will be the only cause of failure. Inasmuch as metal fatigue implies many millions of stress applications successfully endured, we need a quantitative life measure. Common life measures are

- Number of revolutions of the inner ring (outer ring stationary) until the first tangible evidence of fatigue
- Number of hours of use at a standard angular speed until the first tangible evidence of fatigue

The commonly used term is *bearing life,* which is applied to either of the measures just mentioned. It is important to realize, as in all fatigue, life as defined above is a stochastic variable and, as such, has both a distribution and associated statistical parameters. The life measure of an individual bearing is defined as the total number of revolutions (or hours at a constant speed) of bearing operation until the failure criterion is developed. Under ideal conditions, the fatigue failure consists of spalling of the load-carrying surfaces. The American Bearing Manufacturers Association (ABMA) standard states that the failure criterion is the first evidence of fatigue. The fatigue criterion used by the Timken Company laboratories is the spalling or pitting of an area of 0.01 in^2. Timken also observes that the useful life of the bearing may extend considerably beyond this point. This is an operational definition of fatigue failure in rolling bearings.

The *rating life* is a term sanctioned by the ABMA and used by most manufacturers. The rating life of a group of nominally identical ball or roller bearings is defined as the number of revolutions (or hours at a constant speed) that 90 percent of a group of bearings will achieve or exceed before the failure criterion develops. The terms *minimum life,* L_{10} *life,* and B_{10} *life* are also used as synonyms for rating life. The rating life is the 10th percentile location of the bearing group's revolutions-to-failure distribution.

Median life is the 50th percentile life of a group of bearings. The term *average life* has been used as a synonym for median life, contributing to confusion. When many groups of bearings are tested, the median life is between 4 and 5 times the L_{10} life.

Each bearing manufacturer will choose a specific rating life for which load ratings of its bearings are reported. The most commonly used rating life is 10^6 revolutions. The Timken Company is a well-known exception, rating its bearings at 3 000 hours at 500 rev/min, which is $90(10^6)$ revolutions. These levels of rating life are actually quite low for today's bearings, but since rating life is an arbitrary reference point, the traditional values have generally been maintained.

11–3 Bearing Load Life at Rated Reliability

When nominally identical groups are tested to the life-failure criterion at different loads, the data are plotted on a graph as depicted in Fig. 11–4 using a log-log transformation. To establish a single point, load F_1 and the rating life of group one $(L_{10})_1$ are the coordinates that are logarithmically transformed. The reliability associated with this point, and all other points, is 0.90. Thus we gain a glimpse of the load-life function at 0.90 reliability. Using a regression equation of the form

$$FL^{1/a} = \text{constant} \tag{11–1}$$

the result of many tests for various kinds of bearings result in

- $a = 3$ for ball bearings
- $a = 10/3$ for roller bearings (cylindrical and tapered roller)

A *catalog load rating* is defined as the radial load that causes 10 percent of a group of bearings to fail at the bearing manufacturer's rating life. We shall denote the catalog load rating as C_{10}. The catalog load rating is often referred to as a *Basic Dynamic Load Rating,* or sometimes just Basic Load Rating, if the manufacturer's rating life is 10^6 revolutions. The radial load that would be necessary to cause failure at such a low life would be unrealistically high. Consequently, the Basic Load Rating should be viewed as a reference value, and not as an actual load to be achieved by a bearing.

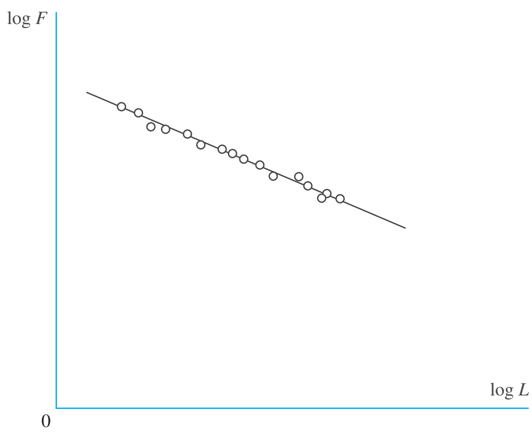


Figure 11–4

Typical bearing load-life log-log curve.

In selecting a bearing for a given application, it is necessary to relate the desired load and life requirements to the published catalog load rating corresponding to the catalog rating life. From Eq. (11–1) we can write

$$F_1 L_1^{1/a} = F_2 L_2^{1/a} \tag{11–2}$$

where the subscripts 1 and 2 can refer to any set of load and life conditions. Letting F_1 and L_1 correlate with the catalog load rating and rating life, and F_2 and L_2 correlate with desired load and life for the application, we can express Eq. (11–2) as

$$F_R L_R^{1/a} = F_D L_D^{1/a} \tag{a}$$

where the units of L_R and L_D are revolutions, and the subscripts R and D stand for Rated and Desired.

It is sometimes convenient to express the life in hours at a given speed. Accordingly, any life L in revolutions can be expressed as

$$L = 60\,\mathcal{L}n \tag{b}$$

where $\mathcal{L}$ is in hours, n is in rev/min, and 60 min/h is the appropriate conversion factor.

Incorporating Eq. (b) into Eq. (a),

$$F_R(\mathcal{L}_R n_R 60)^{1/a} = F_D(\mathcal{L}_D n_D 60)^{1/a} \tag{c}$$

- F_R: catalog rating, lbf or kN
- $\mathcal{L}_R$: rating life in hours
- n_R: rating speed, rev/min
- F_D: desired radial load, lbf or kN
- $\mathcal{L}_D$: desired life, hours
- n_D: desired speed, rev/min

Solving Eq. (c) for F_R, and noting that it is simply an alternate notation for the catalog load rating C_{10}, we obtain an expression for a catalog load rating as a function of the desired load, desired life, and catalog rating life.

$$C_{10} = F_R = F_D\left(\frac{L_D}{L_R}\right)^{1/a} = F_D\left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60}\right)^{1/a} \tag{11–3}$$

It is sometimes convenient to define $x_D = L_D/L_R$ as a dimensionless *multiple of rating life.*

EXAMPLE 11–1 Consider SKF, which rates its bearings for 1 million revolutions. If you desire a life of 5000 h at 1725 rev/min with a load of 400 lbf with a reliability of 90 percent, for which catalog rating would you search in an SKF catalog?

Solution The rating life is $L_{10} = L_R = \mathcal{L}_R n_R 60 = 10^6$ revolutions. From Eq. (11–3),

Answer

$$C_{10} = F_D\left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60}\right)^{1/a} = 400\left[\frac{5000(1725)60}{10^6}\right]^{1/3} = 3211 \text{ lbf} = 14.3 \text{ kN}$$

11–4 Reliability versus Life—The Weibull Distribution

At constant load, the life measure distribution of rolling-contact bearings is right skewed. Because of its robust ability to adjust to varying amounts of skewness, the *three-parameter Weibull distribution* is used exclusively for expressing the reliability of rolling-contact bearings. Unlike the development of the normal distribution in Sec. 1–12, we will begin with the definition of the reliability, R, for a Weibull distribution of the life measure, x, as

$$R = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right] \tag{11–4}$$

where the three parameters are[1]

x_0 = guaranteed, or "minimum," value of x

θ = characteristic parameter. For rolling-contact bearings, this corresponds to the 63.2121 percentile value of x

b = shape parameter that controls the skewness. For rolling-contact bearings, $b \approx 1.5$

The life measure is expressed in dimensionless form as $x = L/L_{10}$.

From Eq. (1–8), $R = 1 - p$, where p is the probability of a value of x occurring between $-\infty$ and x, and is the integral of the probability distribution, $f(x)$, between those limits. Accordingly, $f(x) = -dR/dx$. Thus, from the derivative of Eq. (11–4), the Weibull probability density function, $f(x)$, is given by

$$f(x) = \begin{cases} \dfrac{b}{\theta - x_0}\left(\dfrac{x - x_0}{\theta - x_0}\right)^{b-1} \exp\left[-\left(\dfrac{x - x_0}{\theta - x_0}\right)^b\right] & x \geq x_0 \geq 0 \\ 0 & x < x_0 \end{cases} \tag{11–5}$$

The mean and standard deviation of $f(x)$ are

$$\mu_x = x_0 + (\theta - x_0)\Gamma(1 + 1/b) \tag{11–6}$$

$$\hat{\sigma}_x = (\theta - x_0)\sqrt{\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)} \tag{11–7}$$

where Γ is the *gamma function,* and is found tabulated in Table A–34.

Given a specific required reliability, solving Eq. (11–4) for x yields

$$x = x_0 + (\theta - x_0)\left(\ln\frac{1}{R}\right)^{1/b} \tag{11–8}$$

EXAMPLE 11–2

Construct the distributional properties of a 02–30 mm deep-groove ball bearing if the Weibull parameters are $x_0 = 0.020$, $\theta = 4.459$, and $b = 1.483$.

Find the mean, median, 10th percentile life, standard deviation, and coefficient of variation.

[1]To estimate the Weibull parameters from data, see J. E. Shigley and C. R. Mischke, *Mechanical Engineering Design,* 5th ed., McGraw-Hill, New York, 1989, Sec. 4–12, Ex. 4–10.

Solution From Eq. (11–6) and interpolating Table A–34, the mean dimensionless life is

Answer
$$\mu_x = x_0 + (\theta - x_0)\Gamma(1 + 1/b)$$
$$= 0.020 + (4.459 - 0.020)\Gamma(1 + 1/1.483)$$
$$= 0.020 + 4.439\Gamma(1.67431) = 0.020 + 4.439(0.9040) = 4.033$$

This says that the average bearing life is 4.033 L_{10}.

The median dimensionless life corresponds to $R = 0.50$, or L_{50}, and from Eq. (11–8) is

Answer
$$x_{0.50} = x_0 + (\theta - x_0)\left(\ln\frac{1}{0.50}\right)^{1/b}$$
$$= 0.020 + (4.459 - 0.020)\left(\ln\frac{1}{0.50}\right)^{1/1.483} = 3.487$$

or, $L = 3.487\ L_{10}$.

The 10th percentile value of the dimensionless life x is

Answer
$$x_{0.10} = 0.020 + (4.459 - 0.020)\left(\ln\frac{1}{0.90}\right)^{1/1.483} \approx 1 \qquad \text{(as it should be)}$$

The standard deviation of the dimensionless life, given by Eq. (11–7), is

Answer
$$\hat{\sigma}_x = (\theta - x_0)\sqrt{\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)}$$
$$= (4.459 - 0.020)\sqrt{\Gamma(1 + 2/1.483) - \Gamma^2(1 + 1/1.483)}$$
$$= 4.439\sqrt{\Gamma(2.349) - \Gamma^2(1.674)} = 4.439\sqrt{1.2023 - 0.9040^2}$$
$$= 2.755$$

The coefficient of variation of the dimensionless life is

Answer
$$C_x = \frac{\hat{\sigma}_x}{\mu_x} = \frac{2.755}{4.033} = 0.683$$

11–5 Relating Load, Life, and Reliability

This is the designer's problem. The desired load is not the manufacturer's test load or catalog entry. The desired speed is different from the vendor's test speed, and the reliability expectation is typically much higher than the 0.90 accompanying the catalog entry. Figure 11–5 shows the situation. The catalog information is plotted as point A, whose coordinates are (the logs of) C_{10} and $x_{10} = L_{10}/L_{10} = 1$, a point on the 0.90 reliability contour. The design point is at D, with the coordinates (the logs of) F_D and x_D, a point that is on the $R = R_D$ reliability contour. The designer must move from point D to point A via point B as follows. Along a constant reliability contour (BD), Eq. (11–2) applies:

$$F_B x_B^{1/a} = F_D x_D^{1/a}$$

from which

$$F_B = F_D\left(\frac{x_D}{x_B}\right)^{1/a} \qquad (a)$$

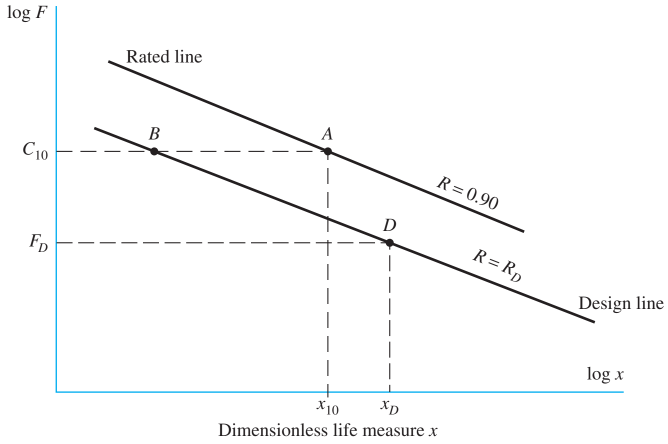


Figure 11–5

Constant reliability contours. Point A represents the catalog rating C_{10} at $x = L/L_{10} = 1$. Point B is on the target reliability design line R_D, with a load of C_{10}. Point D is a point on the desired reliability contour exhibiting the design life $x_D = L_D/L_{10}$ at the design load F_D.

Along a constant load line (AB), Eq. (11–4) applies:

$$R_D = \exp\left[-\left(\frac{x_B - x_0}{\theta - x_0}\right)^b\right]$$

Solving for x_B gives

$$x_B = x_0 + (\theta - x_0)\left(\ln\frac{1}{R_D}\right)^{1/b}$$

Now substitute this in Eq. (a) to obtain

$$F_B = F_D\left(\frac{x_D}{x_B}\right)^{1/a} = F_D\left[\frac{x_D}{x_0 + (\theta - x_0)[\ln(1/R_D)]^{1/b}}\right]^{1/a}$$

Noting that $F_B = C_{10}$, and including an application factor a_f with the design load,

$$C_{10} = a_f F_D\left[\frac{x_D}{x_0 + (\theta - x_0)[\ln(1/R_D)]^{1/b}}\right]^{1/a} \tag{11–9}$$

The application factor serves as a factor of safety to increase the design load to take into account overload, dynamic loading, and uncertainty. Typical load application factors for certain types of applications will be discussed shortly.

Eq. (11–9) can be simplified slightly for calculator entry by noting that

$$\ln\frac{1}{R_D} = \ln\frac{1}{1 - p_f} = \ln(1 + p_f + \cdots) \approx p_f = 1 - R_D$$

where p_f is the probability for failure. Equation (11–9) can be written as

$$C_{10} \approx a_f F_D\left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}}\right]^{1/a} \qquad R \geq 0.90 \tag{11–10}$$

Either Eq. (11–9) or Eq. (11–10) may be used to convert from a design situation with a desired load, life, and reliability to a catalog load rating based on a rating life at 90 percent reliability. Note that when $R_D = 0.90$, the denominator is equal to one, and the equation reduces to Eq. (11–3). The Weibull parameters are usually provided in the manufacturer's catalog. Typical values are given on p. 601 at the beginning of the end-of-chapter problems.

EXAMPLE 11–3 The design load on a ball bearing is 413 lbf and an application factor of 1.2 is appropriate. The speed of the shaft is to be 300 rev/min, the life to be 30 kh with a reliability of 0.99. What is the C_{10} catalog entry to be sought (or exceeded) when searching for a deep-groove bearing in a manufacturer's catalog on the basis of 10^6 revolutions for rating life? The Weibull parameters are $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$.

Solution

$$x_D = \frac{L_D}{L_R} = \frac{60\mathcal{L}_D n_D}{L_{10}} = \frac{60(30\,000)300}{10^6} = 540$$

Thus, the design life is 540 times the L_{10} life. For a ball bearing, $a = 3$. Then, from Eq. (11–10),

Answer

$$C_{10} = (1.2)(413)\left[\frac{540}{0.02 + 4.439(1 - 0.99)^{1/1.483}}\right]^{1/3} = 6696 \text{ lbf}$$

Shafts generally have two bearings. Often these bearings are different. If the bearing reliability of the shaft with its pair of bearings is to be R, then R is related to the individual bearing reliabilities R_A and R_B, using Eq. (1–9), as

$$R = R_A R_B$$

First, we observe that if the product $R_A R_B$ equals R, then, in general, R_A and R_B are both greater than R. Since the failure of either or both of the bearings results in the shutdown of the shaft, then A or B or both can create a failure. Second, in sizing bearings one can begin by making R_A and R_B equal to the square root of the reliability goal, $\sqrt{R}$. In Ex. 11–3, if the bearing was one of a pair, the reliability goal would be $\sqrt{0.99}$, or 0.995. The bearings selected are discrete in their reliability property in your problem, so the selection procedure "rounds up," and the overall reliability exceeds the goal R. Third, it may be possible, if $R_A > \sqrt{R}$, to round down on B yet have the product $R_A R_B$ still exceed the goal R.

11–6 Combined Radial and Thrust Loading

A ball bearing is capable of resisting radial loading and a thrust loading. Furthermore, these can be combined. Consider F_a and F_r to be the axial thrust and radial loads, respectively, and F_e to be the *equivalent radial load* that does the same damage as the combined radial and thrust loads together. A rotation factor V is defined such that $V = 1$ when the inner ring rotates and $V = 1.2$ when the outer ring rotates. Two dimensionless groups can now be formed: $F_e/(VF_r)$ and $F_a/(VF_r)$. When these two dimensionless groups are plotted as in Fig. 11–6, the data fall in a gentle curve that is well approximated by two straight-line segments. The abscissa e is defined by the intersection of the two lines. The equations for the two lines shown in Fig. 11–6 are

$$\frac{F_e}{VF_r} = 1 \qquad \text{when } \frac{F_a}{VF_r} \le e \tag{11–11a}$$

$$\frac{F_e}{VF_r} = X + Y\frac{F_a}{VF_r} \qquad \text{when } \frac{F_a}{VF_r} > e \tag{11–11b}$$

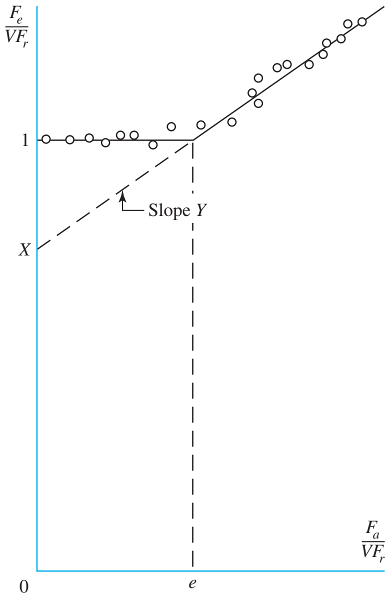


Figure 11–6

The relationship of dimensionless group $F_e/(VF_r)$ and $F_a/(VF_r)$ and the straight-line segments representing the data.

where, as shown, X is the ordinate intercept and Y is the slope of the line for $F_a/(VF_r) > e$. It is common to express Eqs. (11–11a) and (11–11b) as a single equation,

$$F_e = X_i VF_r + Y_i F_a \tag{11–12}$$

where $i = 1$ when $F_a/(VF_r) \leq e$ and $i = 2$ when $F_a/(VF_r) > e$. The X and Y factors depend upon the geometry and construction of the specific bearing. Table 11–1 lists representative values of X_1, Y_1, X_2, and Y_2 as a function of e, which in turn is a function of F_a/C_0, where C_0 is the basic static load rating. The *basic static load rating* is the load that will produce a total permanent deformation in the raceway and rolling

Table 11–1

Equivalent Radial Load Factors for Ball Bearings

		$F_a/(VF_r) \leq e$		$F_a/(VF_r) > e$	
F_a/C_0	e	X_1	Y_1	X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

*Use 0.014 if $F_a/C_0 < 0.014$.

element at any contact point of 0.0001 times the diameter of the rolling element. The basic static load rating is typically tabulated, along with the basic dynamic load rating C_{10}, in bearing manufacturers' publications. See Table 11–2, for example.

In these equations, the rotation factor V is intended to correct for the rotating-ring conditions. The factor of 1.2 for outer-ring rotation is simply an acknowledgment that the fatigue life is reduced under these conditions. Self-aligning bearings are an exception: they have $V = 1$ for rotation of either ring.

Since straight or cylindrical roller bearings will take no axial load, or very little, the Y factor is always zero.

The ABMA has established standard boundary dimensions for bearings, which define the bearing bore, the outside diameter (OD), the width, and the fillet sizes on the shaft and housing shoulders. The basic plan covers all ball and straight roller bearings in the metric sizes. The plan is quite flexible in that, for a given bore, there is an assortment of widths and outside diameters. Furthermore, the outside diameters selected are such that, for a particular outside diameter, one can usually find a variety of bearings having different bores and widths.

Table 11–2

Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

Bore, mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm		Load Ratings, kN			
						Deep Groove		Angular Contact	
				d_S	d_H	C_{10}	C_0	C_{10}	C_0
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

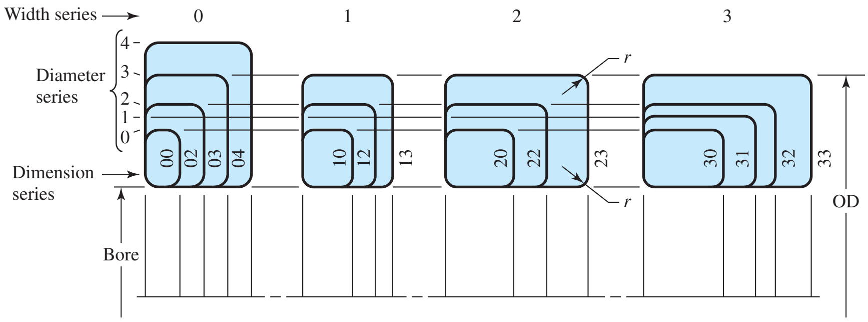


Figure 11–7

The basic ABMA plan for boundary dimensions. These apply to ball bearings, straight roller bearings, and spherical roller bearings, but not to inch-series ball bearings or tapered roller bearings. The contour of the corner is not specified. It may be rounded or chamfered, but it must be small enough to clear the fillet radius specified in the standards.

This basic ABMA plan is illustrated in Fig. 11–7. The bearings are identified by a two-digit number called the *dimension-series code*. The first number in the code is from the *width series,* 0, 1, 2, 3, 4, 5, and 6. The second number is from the *diameter series* (outside), 8, 9, 0, 1, 2, 3, and 4. Figure 11–7 shows the variety of bearings that may be obtained with a particular bore. Since the dimension-series code does not reveal the dimensions directly, it is necessary to resort to tabulations. The 02 series is used here as an example of what is available. See Table 11–2.

The housing and shaft shoulder diameters listed in the tables should be used whenever possible to secure adequate support for the bearing and to resist the maximum thrust loads (Fig. 11–8). Table 11–3 lists the dimensions and load ratings of some straight roller bearings.

To assist the designer in the selection of bearings, most of the manufacturers' handbooks contain data on bearing life for many classes of machinery, as well as information on load-application factors. Such information has been accumulated the hard way, that is, by experience, and the beginner designer should utilize this information until he or she gains enough experience to know when deviations are possible. Table 11–4 contains recommendations on bearing life for some classes of machinery. The load-application factors in Table 11–5 serve the same purpose as factors of safety; use them to increase the equivalent load before selecting a bearing.

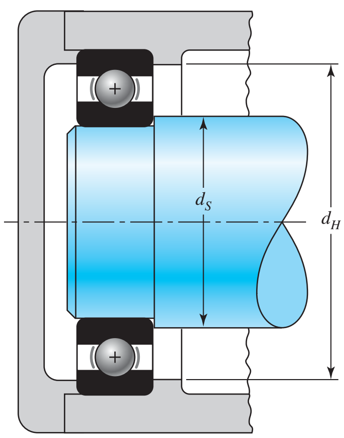


Figure 11–8

Shaft and housing shoulder diameters d_S and d_H should be adequate to ensure good bearing support.

Table 11–3

Dimensions and Basic Load Ratings for Cylindrical Roller Bearings

	02-Series				03-Series			
Bore, mm	OD, mm	Width, mm	Load Rating, kN C_{10}	Load Rating, kN C_0	OD, mm	Width, mm	Load Rating, kN C_{10}	Load Rating, kN C_0
25	52	15	16.8	8.8	62	17	28.6	15.0
30	62	16	22.4	12.0	72	19	36.9	20.0
35	72	17	31.9	17.6	80	21	44.6	27.1
40	80	18	41.8	24.0	90	23	56.1	32.5
45	85	19	44.0	25.5	100	25	72.1	45.4
50	90	20	45.7	27.5	110	27	88.0	52.0
55	100	21	56.1	34.0	120	29	102	67.2
60	110	22	64.4	43.1	130	31	123	76.5
65	120	23	76.5	51.2	140	33	138	85.0
70	125	24	79.2	51.2	150	35	151	102
75	130	25	93.1	63.2	160	37	183	125
80	140	26	106	69.4	170	39	190	125
85	150	28	119	78.3	180	41	212	149
90	160	30	142	100	190	43	242	160
95	170	32	165	112	200	45	264	189
100	180	34	183	125	215	47	303	220
110	200	38	229	167	240	50	391	304
120	215	40	260	183	260	55	457	340
130	230	40	270	193	280	58	539	408
140	250	42	319	240	300	62	682	454
150	270	45	446	260	320	65	781	502

Table 11–4

Bearing-Life Recommendations for Various Classes of Machinery

Type of Application	Life, kh
Instruments and apparatus for infrequent use	Up to 0.5
Aircraft engines	0.5–2
Machines for short or intermittent operation where service interruption is of minor importance	4–8
Machines for intermittent service where reliable operation is of great importance	8–14
Machines for 8-h service that are not always fully utilized	14–20
Machines for 8-h service that are fully utilized	20–30
Machines for continuous 24-h service	50–60
Machines for continuous 24-h service where reliability is of extreme importance	100–200

Table 11–5

Load-Application Factors

Type of Application	Load Factor
Precision gearing	1.0–1.1
Commercial gearing	1.1–1.3
Applications with poor bearing seals	1.2
Machinery with no impact	1.0–1.2
Machinery with light impact	1.2–1.5
Machinery with moderate impact	1.5–3.0

EXAMPLE 11–4 An SKF 6210 angular-contact ball bearing has an axial load F_a of 400 lbf and a radial load F_r of 500 lbf applied with the outer ring stationary. The basic static load rating C_0 is 4450 lbf and the basic load rating C_{10} is 7900 lbf. Estimate the $\mathcal{L}_{10}$ life at a speed of 720 rev/min.

Solution $V = 1$ and $F_a/C_0 = 400/4450 = 0.090$. Interpolate for e in Table 11–1:

F_a/C_0	e	
0.084	0.28	
0.090	e	from which $e = 0.285$
0.110	0.30	

$F_a/(VF_r) = 400/[(1)500] = 0.8 > 0.285$. Thus, interpolate for Y_2:

F_a/C_0	Y_2	
0.084	1.55	
0.090	Y_2	from which $Y_2 = 1.527$
0.110	1.45	

From Eq. (11–12),

$$F_e = X_2VF_r + Y_2F_a = 0.56(1)500 + 1.527(400) = 890.8 \text{ lbf}$$

With $\mathcal{L}_D = \mathcal{L}_{10}$ and $F_D = F_e$, solving Eq. (11–3) for $\mathcal{L}_{10}$ gives

Answer

$$\mathcal{L}_{10} = \frac{60\mathcal{L}_R n_R}{60 n_D}\left(\frac{C_{10}}{F_e}\right)^a = \frac{10^6}{60(720)}\left(\frac{7900}{890.8}\right)^3 = 16\,150 \text{ h}$$

We now know how to combine a steady radial load and a steady thrust load into an equivalent steady radial load F_e that inflicts the same damage per revolution as the radial-thrust combination.

11–7 Variable Loading

Bearing loads are frequently variable and occur in some identifiable patterns:

- Piecewise constant loading in a cyclic pattern
- Continuously variable loading in a repeatable cyclic pattern
- Random variation

Equation (11–1) can be written as

$$F^a L = \text{constant} = K \tag{a}$$

Note that F may already be an equivalent steady radial load for a radial–thrust load combination. Figure 11–9 is a plot of F^a as ordinate and L as abscissa for Eq. (a). If a load level of F_1 is selected and run to the failure criterion, then the area under the F_1-L_1 trace is numerically equal to K. The same is true for a load level F_2; that is, the area under the F_2-L_2 trace is numerically equal to K. The linear damage theory says that in the case of load level F_1, the area from $L = 0$ to $L = L_A$ does damage measured by $F_1^a L_A = D$.

Consider the piecewise continuous cycle depicted in Fig. 11–10. The loads F_{ei} are equivalent steady radial loads for combined radial–thrust loads. The damage done by loads F_{e1}, F_{e2}, and F_{e3} is

$$D = F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3 \tag{b}$$

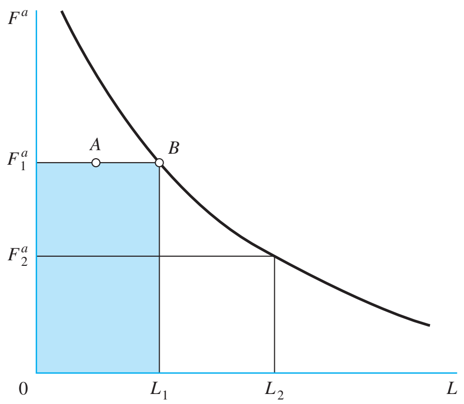


Figure 11–9

Plot of F^a as ordinate and L as abscissa for F^aL = constant. The linear damage hypothesis says that in the case of load F_1, the area under the curve from $L = 0$ to $L = L_A$ is a measure of the damage $D = F_1^a L_A$. The complete damage to failure is measured by $C_{10}^a L_B$.

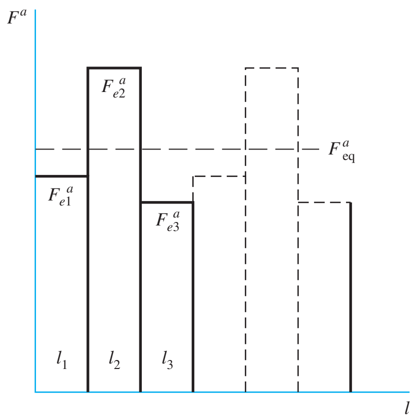


Figure 11–10

A three-part piecewise-continuous periodic loading cycle involving loads F_{e1}, F_{e2}, and F_{e3}. F_{eq} is the equivalent steady load inflicting the same damage when run for $l_1 + l_2 + l_3$ revolutions, doing the same damage D per period.

where l_i is the number of revolutions at life L_i. The equivalent steady load F_{eq} when run for $l_1 + l_2 + l_3$ revolutions does the same damage D. Thus

$$D = F_{eq}^a(l_1 + l_2 + l_3) \tag{c}$$

Equating Eqs. (b) and (c), and solving for F_{eq}, we get

$$F_{eq} = \left[\frac{F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3}{l_1 + l_2 + l_3}\right]^{1/a} = \left[\sum f_i F_{ei}^a\right]^{1/a} \tag{11–13}$$

where f_i is the fraction of revolution run up under load F_{ei}. Since l_i can be expressed as $n_i t_i$, where n_i is the rotational speed at load F_{ei} and t_i is the duration of that speed, then it follows that

$$F_{eq} = \left[\frac{\sum n_i t_i F_{ei}^a}{\sum n_i t_i}\right]^{1/a} \tag{11–14}$$

The character of the individual loads can change, so an application factor (a_f) can be prefixed to each F_{ei} as $(a_{fi} F_{ei})^a$; then Eq. (11–13) can be written

$$F_{eq} = \left[\sum f_i (a_{fi} F_{ei})^a\right]^{1/a} \qquad L_{eq} = \frac{K}{F_{eq}^a} \tag{11–15}$$

EXAMPLE 11–5 A ball bearing is run at four piecewise continuous steady loads as shown in the following table. Columns (1), (2), and (5) to (8) are given.

(1) Time Fraction	(2) Speed, rev/min	(3) Product, Column (1) × (2)	(4) Turns Fraction, (3)/Σ(3)	(5) F_{ri}, lbf	(6) F_{ai}, lbf	(7) F_{ei}, lbf	(8) a_{fi}	(9) $a_{fi}F_{ei}$, lbf
0.1	2000	200	0.077	600	300	794	1.10	873
0.1	3000	300	0.115	300	300	626	1.25	795
0.3	3000	900	0.346	750	300	878	1.10	966
0.5	2400	1200	0.462	375	300	668	1.25	835
		2600	1.000					

Columns 1 and 2 are multiplied to obtain column 3. The column 3 entry is divided by the sum of column 3, 2600, to give column 4. Columns 5, 6, and 7 are the radial, axial, and equivalent loads respectively. Column 8 is the appropriate application factor. Column 9 is the product of columns 7 and 8.

Solution From Eq. (11–13), with $a = 3$, the equivalent radial load F_e is

Answer $$F_e = [0.077(873)^3 + 0.115(795)^3 + 0.346(966)^3 + 0.462(835)^3]^{1/3} = 884 \text{ lbf}$$

Sometimes the question after several levels of loading is: How much life is left if the next level of stress is held until failure? Failure occurs under the linear damage hypothesis when the damage D equals the constant $K = F^a L$. Taking the first form of Eq. (11–13), we write

$$F_{\text{eq}}^a L_{\text{eq}} = F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3$$

and note that

$$K = F_{e1}^a L_1 = F_{e2}^a L_2 = F_{e3}^a L_3$$

and K also equals

$$K = F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3 = \frac{K}{L_1} l_1 + \frac{K}{L_2} l_2 + \frac{K}{L_3} l_3 = K \sum \frac{l_i}{L_i}$$

From the outer parts of the preceding equation we obtain

$$\sum \frac{l_i}{L_i} = 1 \tag{11–16}$$

This equation was advanced by Palmgren in 1924, and again by Miner in 1945. See Eq. (6–58), p. 331.

The second kind of load variation mentioned is continuous, periodic variation, depicted by Fig. 11–11. The differential damage done by F^a during rotation through the angle $d\theta$ is

$$dD = F^a \, d\theta$$

An example of this would be a cam whose bearings rotate with the cam through the angle $d\theta$. The total damage during a complete cam rotation is given by

$$D = \int dD = \int_0^{\phi} F^a \, d\theta = F_{\text{eq}}^a \phi$$

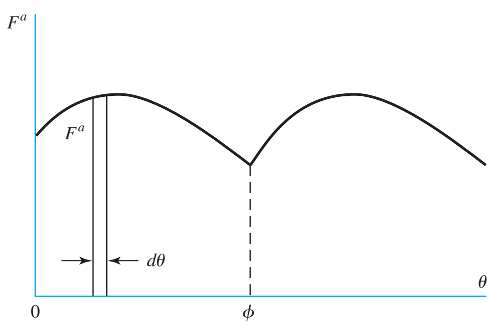
from which, solving for the equivalent load, we obtain

$$F_{\text{eq}} = \left[\frac{1}{\phi} \int_0^{\phi} F^a \, d\theta \right]^{1/a} \qquad L_{\text{eq}} = \frac{K}{F_{\text{eq}}^a} \tag{11–17}$$

The value of ϕ is often 2π, although other values occur. Numerical integration is often useful to carry out the indicated integration, particularly when a is not an integer and trigonometric functions are involved. We have now learned how to find the steady equivalent load that does the same damage as a continuously varying cyclic load.

Figure 11–11

A continuous load variation of a cyclic nature whose period is ϕ.



EXAMPLE 11–6 The operation of a particular rotary pump involves a power demand of $P = \bar{P} + A' \sin\theta$ where $\bar{P}$ is the average power. The bearings feel the same variation as $F = \bar{F} + A\sin\theta$. Develop an application factor a_f for this application of ball bearings.

Solution From Eq. (11–17), with $a = 3$,

$$F_{eq} = \left(\frac{1}{2\pi}\int_0^{2\pi} F^a\, d\theta\right)^{1/a} = \left(\frac{1}{2\pi}\int_0^{2\pi} (\bar{F} + A\sin\theta)^3\, d\theta\right)^{1/3}$$

$$= \left[\frac{1}{2\pi}\left(\int_0^{2\pi} \bar{F}^3\, d\theta + 3\bar{F}^2 A\int_0^{2\pi} \sin\theta\, d\theta + 3\bar{F}A^2\int_0^{2\pi} \sin^2\theta\, d\theta\right.\right.$$

$$\left.\left. + A^3\int_0^{2\pi} \sin^3\theta\, d\theta\right)\right]^{1/3}$$

$$F_{eq} = \left[\frac{1}{2\pi}(2\pi\bar{F}^3 + 0 + 3\pi\bar{F}A^2 + 0)\right]^{1/3} = \bar{F}\left[1 + \frac{3}{2}\left(\frac{A}{\bar{F}}\right)^2\right]^{1/3}$$

In terms of $\bar{F}$, the application factor is

Answer

$$a_f = \left[1 + \frac{3}{2}\left(\frac{A}{\bar{F}}\right)^2\right]^{1/3}$$

We can present the result in tabular form:

$A/\bar{F}$	a_f
0	1
0.2	1.02
0.4	1.07
0.6	1.15
0.8	1.25
1.0	1.36

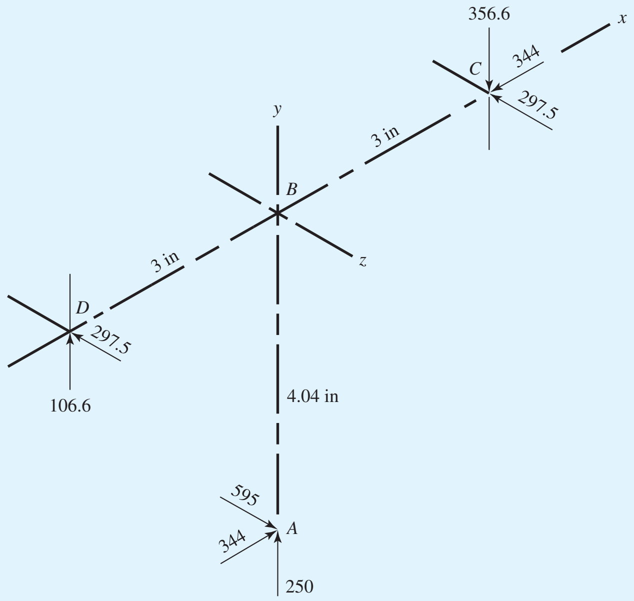
11–8 Selection of Ball and Cylindrical Roller Bearings

We have enough information concerning the loading of rolling-contact ball and roller bearings to develop the steady equivalent radial load that will do as much damage to the bearing as the existing loading. Now let's put it to work.

EXAMPLE 11–7 The second shaft on a parallel-shaft 25-hp foundry crane speed reducer contains a helical gear with a pitch diameter of 8.08 in. Helical gears transmit components of force in the tangential, radial, and axial directions (see Chap. 13). The components of the gear force transmitted to the second shaft are shown in Fig. 11–12, at point A. The bearing

Figure 11–12

Forces in pounds applied to the second shaft of the helical gear speed reducer of Ex. 11–7.



reactions at C and D, assuming simple-supports, are also shown. A ball bearing is to be selected for location C to accept the thrust, and a cylindrical roller bearing is to be utilized at location D. The life goal of the speed reducer is 10 kh, with a reliability factor for the ensemble of all four bearings (both shafts) to equal or exceed 0.96 for the Weibull parameters of Ex. 11–3. The application factor is to be 1.2.

(*a*) Select the roller bearing for location D.

(*b*) Select the ball bearing (angular contact) for location C, assuming the inner ring rotates.

Solution The torque transmitted is $T = 595(4.04) = 2404$ lbf · in. The speed at the rated horsepower, given by Eq. (3–42), p. 116, is

$$n_D = \frac{63\,025H}{T} = \frac{63\,025(25)}{2404} = 655.4 \text{ rev/min}$$

The radial load at D is $\sqrt{106.6^2 + 297.5^2} = 316.0$ lbf, and the radial load at C is $\sqrt{356.6^2 + 297.5^2} = 464.4$ lbf. The individual bearing reliabilities, if equal, must be at least $\sqrt[4]{0.96} = 0.98985 \approx 0.99$. The dimensionless design life for both bearings is

$$x_D = \frac{L_D}{L_{10}} = \frac{60\mathscr{L}_D n_D}{L_{10}} = \frac{60(10\,000)655.4}{10^6} = 393.2$$

(*a*) From Eq. (11–10), the Weibull parameters of Ex. 11–3, an application factor of 1.2, and $a = 10/3$ for the roller bearing at D, the catalog rating should be equal to or greater than

$$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a}$$

$$= 1.2(316.0) \left[\frac{393.2}{0.02 + 4.439(1 - 0.99)^{1/1.483}} \right]^{3/10} = 3591 \text{ lbf} = 16.0 \text{ kN}$$

Answer The absence of a thrust component makes the selection procedure simple. Choose a 02-25 mm series, or a 03-25 mm series cylindrical roller bearing from Table 11–3.

(*b*) The ball bearing at C involves a thrust component. This selection procedure requires an iterative procedure. Assuming $F_a/(VF_r) > e$,

1. Choose Y_2 from Table 11–1.
2. Find C_{10}.
3. Tentatively identify a suitable bearing from Table 11–2, note C_0.
4. Using F_a/C_0 enter Table 11–1 to obtain a new value of Y_2.
5. Find C_{10}.
6. If the same bearing is obtained, stop.
7. If not, take next bearing and go to step 4.

As a first approximation, take the middle entry from Table 11–1:

$$X_2 = 0.56 \qquad Y_2 = 1.63.$$

From Eq. (11–12), with $V = 1$,

$$F_e = XVF_r + YF_a = 0.56(1)(464.4) + 1.63(344) = 821 \text{ lbf} = 3.65 \text{ kN}$$

From Eq. (11–10), with $a = 3$,

$$C_{10} = 1.2(3.65)\left[\frac{393.2}{0.02 + 4.439(1 - 0.99)^{1/1.483}}\right]^{1/3} = 53.2 \text{ kN}$$

From Table 11–2, angular-contact bearing 02-60 mm has $C_{10} = 55.9$ kN. C_0 is 35.5 kN. Step 4 becomes, with F_a in kN,

$$\frac{F_a}{C_0} = \frac{344(4.45)10^{-3}}{35.5} = 0.0431$$

which makes e from Table 11–1 approximately 0.24. Now $F_a/(VF_r) = 344/[(1)\ 464.4] = 0.74$, which is greater than 0.24, so we find Y_2 by interpolation:

F_a/C_0	Y_2	
0.042	1.85	
0.043	Y_2	from which $Y_2 = 1.84$
0.056	1.71	

From Eq. (11–12),

$$F_e = 0.56(1)(464.4) + 1.84(344) = 893 \text{ lbf} = 3.97 \text{ kN}$$

The prior calculation for C_{10} changes only in F_e, so

$$C_{10} = \frac{3.97}{3.65}53.2 = 57.9 \text{ kN}$$

From Table 11–2 an angular contact bearing 02-65 mm has $C_{10} = 63.7$ kN and C_0 of 41.5 kN. Again,

$$\frac{F_a}{C_0} = \frac{344(4.45)10^{-3}}{41.5} = 0.0369$$

making e approximately 0.23. Now from before, $F_a/(VF_r) = 0.74$, which is greater than 0.23. We find Y_2 again by interpolation:

F_a/C_0	Y_2	
0.028	1.99	
0.0369	Y_2	from which $Y_2 = 1.90$
0.042	1.85	

From Eq. (11–12),

$$F_e = 0.56(1)(464.4) + 1.90(344) = 914 \text{ lbf} = 4.07 \text{ kN}$$

The prior calculation for C_{10} changes only in F_e, so

$$C_{10} = \frac{4.07}{3.65} 53.2 = 59.3 \text{ kN}$$

Answer From Table 11–2 an angular-contact 02-65 mm is still selected, so the iteration is complete.

11–9 Selection of Tapered Roller Bearings

Tapered roller bearings have a number of features that make them complicated. As we address the differences between tapered roller and ball and cylindrical roller bearings, note that the underlying fundamentals are the same, but that there are differences in detail. Moreover, bearing and cup combinations are not necessarily priced in proportion to capacity. Any catalog displays a mix of high-production, low-production, and successful special-order designs. Bearing suppliers have computer programs that will take your problem descriptions, give intermediate design assessment information, and list a number of satisfactory cup-and-cone combinations in order of decreasing cost. Company sales offices provide access to comprehensive engineering services to help designers select and apply their bearings. At a large original equipment manufacturer's plant, there may be a resident bearing company representative.

Bearing suppliers provide a wealth of engineering information and detail in their catalogs and engineering guides, both in print and online. It is strongly recommended that the designer become familiar with the specifics of the supplier. It will usually utilize a similar approach as presented here, but may include various modifying factors for such things as temperature and lubrication. Many of the suppliers will provide online software tools to aid in bearing selection. The engineer will always benefit from a general understanding of the theory utilized in such software tools. Our goal here is to introduce the vocabulary, show congruence to fundamentals that were learned earlier, offer examples, and develop confidence. Finally, problems should reinforce the learning experience.

The four components of a tapered roller bearing assembly are the

- Cone (inner ring)
- Cup (outer ring)
- Tapered rollers
- Cage (spacer-retainer)

The assembled bearing consists of two separable parts: (1) the cone assembly: the cone, the rollers, and the cage; and (2) the cup. Bearings can be made as single-row, two-row, four-row, and thrust-bearing assemblies. Additionally, auxiliary components such as spacers and closures can be used. Figure 11–13 shows the nomenclature of a tapered roller bearing, and the point G through which radial and axial components of load act.

A tapered roller bearing can carry both radial and thrust (axial) loads, or any combination of the two. However, even when an external thrust load is not present, the radial load will induce a thrust reaction within the bearing because of the taper. To avoid the separation of the races and the rollers, this thrust must be resisted by an equal and opposite force. One way of generating this force is to always use at least two tapered roller bearings on a shaft. Two bearings can be mounted with the cone backs facing each other, in a configuration called *direct mounting,* or with the cone fronts facing each other, in what is called *indirect mounting.*

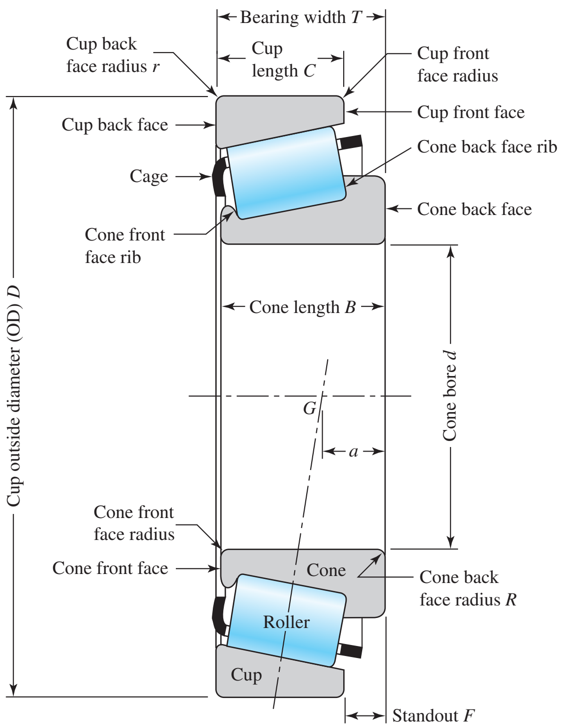


Figure 11–13

Nomenclature of a tapered roller bearing. Point G is the location of the effective load center; use this point to estimate the radial bearing load. *(Courtesy of The Timken Company.)*

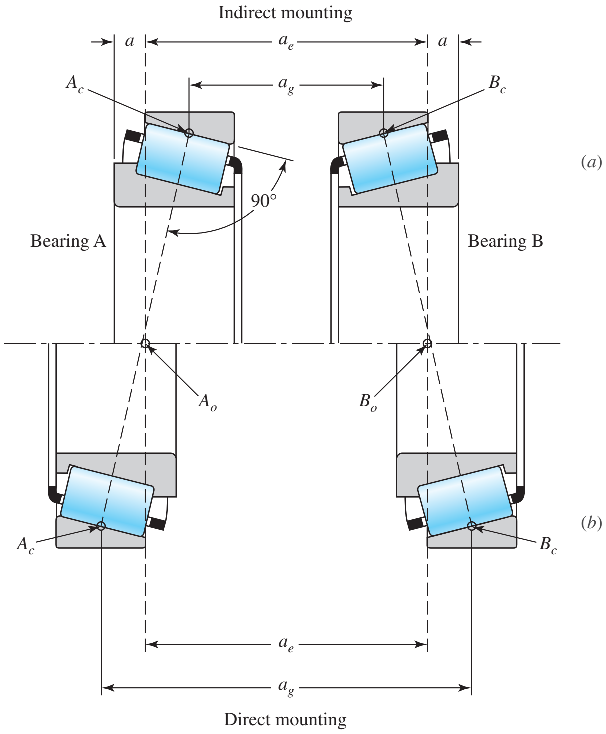


Figure 11–14

Comparison of mounting stability between indirect and direct mountings. *(Courtesy of The Timken Company.)*

Figure 11–14 shows a pair of tapered roller bearings mounted directly (*b*) and indirectly (*a*) with the bearing reaction locations A_0 and B_0 shown for the shaft. For the shaft as a beam, the span is a_e, the effective spread. It is through points A_0 and B_0 that the radial loads act perpendicular to the shaft axis, and the thrust loads act along the shaft axis. The geometric spread a_g for the direct mounting is greater than for the indirect mounting. With indirect mounting the bearings are closer together compared to the direct mounting; however, the system stability is the same (a_e is the same in both cases). Thus direct and indirect mounting involve space and compactness needed or desired, but with the same system stability.

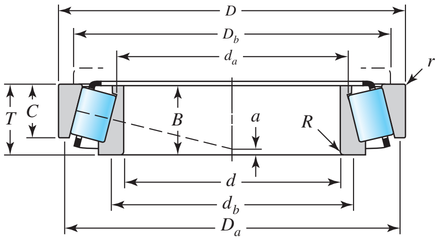
In addition to the usual ratings and geometry information, catalog data for tapered roller bearings will include the location of the effective force center. Two sample pages from a Timken catalog are shown in Fig. 11–15.

A radial load on a tapered roller bearing will induce a thrust reaction. The *load zone* includes about half the rollers and subtends an angle of approximately 180°. Using the symbol F_i for the induced thrust load from a radial load with a 180° load zone, Timken provides the equation

$$F_i = \frac{0.47F_r}{K} \tag{11–18}$$

where the K factor is geometry-specific, and is the ratio of the radial load rating to the thrust load rating. The K factor can be first approximated with 1.5 for a radial bearing and 0.75 for a steep angle bearing in the preliminary selection process. After a possible bearing is identified, the exact value of K for each bearing can be found in the bearing catalog.

SINGLE-ROW STRAIGHT BORE



bore d	outside diameter D	width T	rating at 500 rpm for 3000 hours L_{10}: one-row radial N lbf	rating at 500 rpm for 3000 hours L_{10}: thrust N lbf	fac-tor K	eff. load center a②	part numbers: cone	part numbers: cup	cone: max shaft fillet radius R①	cone: width B	cone: backing shoulder diameters d_b	cone: backing shoulder diameters d_a	cup: max hous-ing fillet radius r①	cup: width C	cup: backing shoulder diameters D_b	cup: backing shoulder diameters D_a
25.000 0.9843	52.000 2.0472	16.250 0.6398	8190 1840	5260 1180	1.56	−3.6 −0.14	◆30205	◆30205	1.0 0.04	15.000 0.5906	30.5 1.20	29.0 1.14	1.0 0.04	13.000 0.5118	46.0 1.81	48.5 1.91
25.000 0.9843	52.000 2.0472	19.250 0.7579	9520 2140	9510 2140	1.00	−3.0 −0.12	◆32205-B	◆32205-B	1.0 0.04	18.000 0.7087	34.0 1.34	31.0 1.22	1.0 0.04	15.000 0.5906	43.5 1.71	49.5 1.95
25.000 0.9843	52.000 2.0472	22.000 0.8661	13200 2980	7960 1790	1.66	−7.6 −0.30	◆33205	◆33205	1.0 0.04	22.000 0.8661	34.0 1.34	30.5 1.20	1.0 0.04	18.000 0.7087	44.5 1.75	49.0 1.93
25.000 0.9843	62.000 2.4409	18.250 0.7185	13000 2930	6680 1500	1.95	−5.1 −0.20	◆30305	◆30305	1.5 0.06	17.000 0.6693	32.5 1.28	30.0 1.18	1.5 0.06	15.000 0.5906	55.0 2.17	57.0 2.24
25.000 0.9843	62.000 2.4409	25.250 0.9941	17400 3910	8930 2010	1.95	−9.7 −0.38	◆32305	◆32305	1.5 0.06	24.000 0.9449	35.0 1.38	31.5 1.24	1.5 0.06	20.000 0.7874	54.0 2.13	57.0 2.24
25.159 0.9905	50.005 1.9687	13.495 0.5313	6990 1570	4810 1080	1.45	−2.8 −0.11	07096	07196	1.5 0.06	14.260 0.5614	31.5 1.24	29.5 1.16	1.0 0.04	9.525 0.3750	44.5 1.75	47.0 1.85
25.400 1.0000	50.005 1.9687	13.495 0.5313	6990 1570	4810 1080	1.45	−2.8 −0.11	07100	07196	1.0 0.04	14.260 0.5614	30.5 1.20	29.5 1.16	1.0 0.04	9.525 0.3750	44.5 1.75	47.0 1.85
25.400 1.0000	50.005 1.9687	13.495 0.5313	6990 1570	4810 1080	1.45	−2.8 −0.11	07100-S	07196	1.5 0.06	14.260 0.5614	31.5 1.24	29.5 1.16	1.0 0.04	9.525 0.3750	44.5 1.75	47.0 1.85
25.400 1.0000	50.292 1.9800	14.224 0.5600	7210 1620	4620 1040	1.56	−3.3 −0.13	L44642	L44610	3.5 0.14	14.732 0.5800	36.0 1.42	29.5 1.16	1.3 0.05	10.668 0.4200	44.5 1.75	47.0 1.85
25.400 1.0000	50.292 1.9800	14.224 0.5600	7210 1620	4620 1040	1.56	−3.3 −0.13	L44643	L44610	1.3 0.05	14.732 0.5800	31.5 1.24	29.5 1.16	1.3 0.05	10.668 0.4200	44.5 1.75	47.0 1.85
25.400 1.0000	51.994 2.0470	15.011 0.5910	6990 1570	4810 1080	1.45	−2.8 −0.11	07100	07204	1.0 0.04	14.260 0.5614	30.5 1.20	29.5 1.16	1.3 0.05	12.700 0.5000	45.0 1.77	48.0 1.89
25.400 1.0000	56.896 2.2400	19.368 0.7625	10900 2450	5740 1290	1.90	−6.9 −0.27	1780	1729	0.8 0.03	19.837 0.7810	30.5 1.20	30.0 1.18	1.3 0.05	15.875 0.6250	49.0 1.93	51.0 2.01
25.400 1.0000	57.150 2.2500	19.431 0.7650	11700 2620	10900 2450	1.07	−3.0 −0.12	M84548	M84510	1.5 0.06	19.431 0.7650	36.0 1.42	33.0 1.30	1.5 0.06	14.732 0.5800	48.5 1.91	54.0 2.13
25.400 1.0000	58.738 2.3125	19.050 0.7500	11600 2610	6560 1470	1.77	−5.8 −0.23	1986	1932	1.3 0.05	19.355 0.7620	32.5 1.28	30.5 1.20	1.3 0.05	15.080 0.5937	52.0 2.05	54.0 2.13
25.400 1.0000	59.530 2.3437	23.368 0.9200	13900 3140	13000 2930	1.07	−5.1 −0.20	M84249	M84210	0.8 0.03	23.114 0.9100	36.0 1.42	32.5 1.27	1.5 0.06	18.288 0.7200	49.5 1.95	56.0 2.20
25.400 1.0000	60.325 2.3750	19.842 0.7812	11000 2480	6550 1470	1.69	−5.1 −0.20	15578	15523	1.3 0.05	17.462 0.6875	32.5 1.28	30.5 1.20	1.5 0.06	15.875 0.6250	51.0 2.01	54.0 2.13
25.400 1.0000	61.912 2.4375	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15101	15243	0.8 0.03	20.638 0.8125	32.5 1.28	31.5 1.24	2.0 0.08	14.288 0.5625	54.0 2.13	58.0 2.28
25.400 1.0000	62.000 2.4409	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15100	15245	3.5 0.14	20.638 0.8125	38.0 1.50	31.5 1.24	1.3 0.05	14.288 0.5625	55.0 2.17	58.0 2.28
25.400 1.0000	62.000 2.4409	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15101	15245	0.8 0.03	20.638 0.8125	32.5 1.28	31.5 1.24	1.3 0.05	14.288 0.5625	55.0 2.17	58.0 2.28

Figure 11–15 *(Continued on next page)*

Catalog entry of single-row straight-bore Timken roller bearings, in part. *(Courtesy of The Timken Company.)*

SINGLE-ROW STRAIGHT BORE

									cone				cup			
bore	outside diameter	width	rating at 500 rpm for 3000 hours L_{10}		fac-tor	eff. load center	part numbers		max shaft fillet radius	width	backing shoulder diameters		max hous-ing fillet radius	width	backing shoulder diameters	
			one-row radial	thrust			cone	cup								
d	D	T	N lbf	N lbf	K	a②			R①	B	d_b	d_a	r①	C	D_b	D_a
25.400 1.0000	**62.000** 2.4409	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15102**	**15245**	**1.5** 0.06	**20.638** 0.8125	**34.0** 1.34	**31.5** 1.24	**1.3** 0.05	**14.288** 0.5625	**55.0** 2.17	**58.0** 2.28
25.400 1.0000	**62.000** 2.4409	**20.638** 0.8125	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15244**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.3** 0.05	**15.875** 0.6250	**55.0** 2.17	**58.0** 2.28
25.400 1.0000	**63.500** 2.5000	**20.638** 0.8125	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15250**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.3** 0.05	**15.875** 0.6250	**56.0** 2.20	**59.0** 2.32
25.400 1.0000	**63.500** 2.5000	**20.638** 0.8125	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15250X**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.5** 0.06	**15.875** 0.6250	**55.0** 2.17	**59.0** 2.32
25.400 1.0000	**64.292** 2.5312	**21.433** 0.8438	**14500** 3250	**13500** 3040	**1.07**	**−3.3** −0.13	**M86643**	**M86610**	**1.5** 0.06	**21.433** 0.8438	**38.0** 1.50	**36.5** 1.44	**1.5** 0.06	**16.670** 0.6563	**54.0** 2.13	**61.0** 2.40
25.400 1.0000	**65.088** 2.5625	**22.225** 0.8750	**13100** 2950	**16400** 3690	**0.80**	**−2.3** −0.09	**23100**	**23256**	**1.5** 0.06	**21.463** 0.8450	**39.0** 1.54	**34.5** 1.36	**1.5** 0.06	**15.875** 0.6250	**53.0** 2.09	**63.0** 2.48
25.400 1.0000	**66.421** 2.6150	**23.812** 0.9375	**18400** 4140	**8000** 1800	**2.30**	**−9.4** −0.37	**2687**	**2631**	**1.3** 0.05	**25.433** 1.0013	**33.5** 1.32	**31.5** 1.24	**1.3** 0.05	**19.050** 0.7500	**58.0** 2.28	**60.0** 2.36
25.400 1.0000	**68.262** 2.6875	**22.225** 0.8750	**15300** 3440	**10900** 2450	**1.40**	**−5.1** −0.20	**02473**	**02420**	**0.8** 0.03	**22.225** 0.8750	**34.5** 1.36	**33.5** 1.32	**1.5** 0.06	**17.462** 0.6875	**59.0** 2.32	**63.0** 2.48
25.400 1.0000	**72.233** 2.8438	**25.400** 1.0000	**18400** 4140	**17200** 3870	**1.07**	**−4.6** −0.18	**HM88630**	**HM88610**	**0.8** 0.03	**25.400** 1.0000	**39.5** 1.56	**39.5** 1.56	**2.3** 0.09	**19.842** 0.7812	**60.0** 2.36	**69.0** 2.72
25.400 1.0000	**72.626** 2.8593	**30.162** 1.1875	**22700** 5110	**13000** 2910	**1.76**	**−10.2** −0.40	**3189**	**3120**	**0.8** 0.03	**29.997** 1.1810	**35.5** 1.40	**35.0** 1.38	**3.3** 0.13	**23.812** 0.9375	**61.0** 2.40	**67.0** 2.64
26.157 1.0298	**62.000** 2.4409	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15103**	**15245**	**0.8** 0.03	**20.638** 0.8125	**33.0** 1.30	**32.5** 1.28	**1.3** 0.05	**14.288** 0.5625	**55.0** 2.17	**58.0** 2.28
26.162 1.0300	**63.100** 2.4843	**23.812** 0.9375	**18400** 4140	**8000** 1800	**2.30**	**−9.4** −0.37	**2682**	**2630**	**1.5** 0.06	**25.433** 1.0013	**34.5** 1.36	**32.0** 1.26	**0.8** 0.03	**19.050** 0.7500	**57.0** 2.24	**59.0** 2.32
26.162 1.0300	**66.421** 2.6150	**23.812** 0.9375	**18400** 4140	**8000** 1800	**2.30**	**−9.4** −0.37	**2682**	**2631**	**1.5** 0.06	**25.433** 1.0013	**34.5** 1.36	**32.0** 1.26	**1.3** 0.05	**19.050** 0.7500	**58.0** 2.28	**60.0** 2.36
26.975 1.0620	**58.738** 2.3125	**19.050** 0.7500	**11600** 2610	**6560** 1470	**1.77**	**−5.8** −0.23	**1987**	**1932**	**0.8** 0.03	**19.355** 0.7620	**32.5** 1.28	**31.5** 1.24	**1.3** 0.05	**15.080** 0.5937	**52.0** 2.05	**54.0** 2.13
† **26.988** † 1.0625	**50.292** 1.9800	**14.224** 0.5600	**7210** 1620	**4620** 1040	**1.56**	**−3.3** −0.13	**L44649**	**L44610**	**3.5** 0.14	**14.732** 0.5800	**37.5** 1.48	**31.0** 1.22	**1.3** 0.05	**10.668** 0.4200	**44.5** 1.75	**47.0** 1.85
† **26.988** † 1.0625	**60.325** 2.3750	**19.842** 0.7812	**11000** 2480	**6550** 1470	**1.69**	**−5.1** −0.20	**15580**	**15523**	**3.5** 0.14	**17.462** 0.6875	**38.5** 1.52	**32.0** 1.26	**1.5** 0.06	**15.875** 0.6250	**51.0** 2.01	**54.0** 2.13
† **26.988** † 1.0625	**62.000** 2.4409	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15106**	**15245**	**0.8** 0.03	**20.638** 0.8125	**33.5** 1.32	**33.0** 1.30	**1.3** 0.05	**14.288** 0.5625	**55.0** 2.17	**58.0** 2.28
† **26.988** † 1.0625	**66.421** 2.6150	**23.812** 0.9375	**18400** 4140	**8000** 1800	**2.30**	**−9.4** −0.37	**2688**	**2631**	**1.5** 0.06	**25.433** 1.0013	**35.0** 1.38	**33.0** 1.30	**1.3** 0.05	**19.050** 0.7500	**58.0** 2.28	**60.0** 2.36
28.575 1.1250	**56.896** 2.2400	**19.845** 0.7813	**11600** 2610	**6560** 1470	**1.77**	**−5.8** −0.23	**1985**	**1930**	**0.8** 0.03	**19.355** 0.7620	**34.0** 1.34	**33.5** 1.32	**0.8** 0.03	**15.875** 0.6250	**51.0** 2.01	**54.0** 2.11
28.575 1.1250	**57.150** 2.2500	**17.462** 0.6875	**11000** 2480	**6550** 1470	**1.69**	**−5.1** −0.20	**15590**	**15520**	**3.5** 0.14	**17.462** 0.6875	**39.5** 1.56	**33.5** 1.32	**1.5** 0.06	**13.495** 0.5313	**51.0** 2.01	**53.0** 2.09
28.575 1.1250	**58.738** 2.3125	**19.050** 0.7500	**11600** 2610	**6560** 1470	**1.77**	**−5.8** −0.23	**1985**	**1932**	**0.8** 0.03	**19.355** 0.7620	**34.0** 1.34	**33.5** 1.32	**1.3** 0.05	**15.080** 0.5937	**52.0** 2.05	**54.0** 2.13
28.575 1.1250	**58.738** 2.3125	**19.050** 0.7500	**11600** 2610	**6560** 1470	**1.77**	**−5.8** −0.23	**1988**	**1932**	**3.5** 0.14	**19.355** 0.7620	**39.5** 1.56	**33.5** 1.32	**1.3** 0.05	**15.080** 0.5937	**52.0** 2.05	**54.0** 2.13
28.575 1.1250	**60.325** 2.3750	**19.842** 0.7812	**11000** 2480	**6550** 1470	**1.69**	**−5.1** −0.20	**15590**	**15523**	**3.5** 0.14	**17.462** 0.6875	**39.5** 1.56	**33.5** 1.32	**1.5** 0.06	**15.875** 0.6250	**51.0** 2.01	**54.0** 2.13
28.575 1.1250	**60.325** 2.3750	**19.845** 0.7813	**11600** 2610	**6560** 1470	**1.77**	**−5.8** −0.23	**1985**	**1931**	**0.5** 0.03	**19.355** 0.7620	**34.0** 1.34	**33.5** 1.32	**1.3** 0.05	**15.875** 0.6250	**52.0** 2.05	**55.0** 2.17

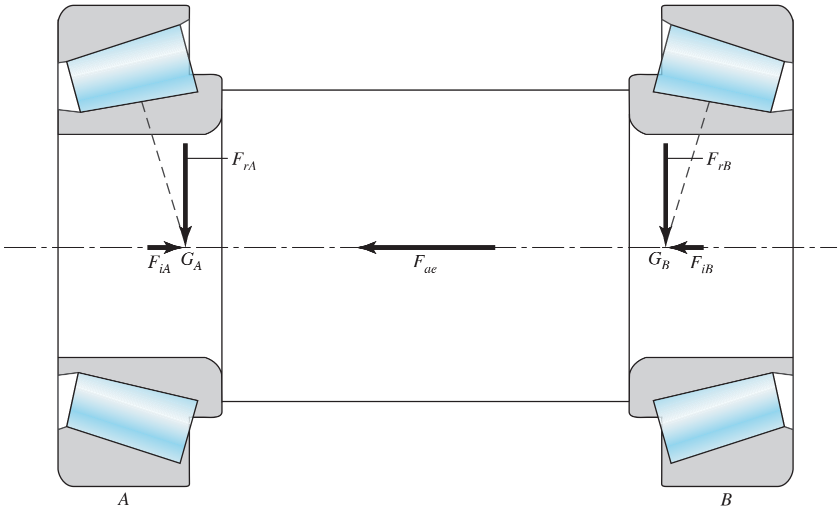
① These maximum fillet radii will be cleared by the bearing corners.
② Minus value indicates center is inside cone backface.
† For standard class **ONLY**, the maximum metric size is a whole mm value.
* For "J" part tolerances—see metric tolerances, page 73, and fitting practice, page 65.
◆ ISO cone and cup combinations are designated with a common part number and should be purchased as an assembly. For ISO bearing tolerances—see metric tolerances, page 73, and fitting practice, page 65.

Figure 11–15

(Continued)

Figure 11–16

Direct-mounted tapered roller bearings, showing radial, induced thrust, and external thrust loads.



A shaft supported by a pair of direct-mounted tapered roller bearings is shown in Fig. 11–16. Force vectors are shown as applied to the shaft. F_{rA} and F_{rB} are the radial loads carried by the bearings, applied at the effective force centers G_A and G_B. The induced loads F_{iA} and F_{iB} due to the effect of the radial loads on the tapered bearings are also shown. Additionally, there may be an externally applied thrust load F_{ae} on the shaft from some other source, such as the axial load on a helical gear. Since the bearings experience both radial and thrust loads, it is necessary to determine equivalent radial loads. Following the form of Eq. (11–12), where $F_e = XVF_r + YF_a$, Timken recommends using $X = 0.4$ and $V = 1$ for all cases, and using the K factor for the specific bearing for Y. This gives an equation of the form

$$F_e = 0.4F_r + KF_a \tag{a}$$

The axial load F_a is the net axial load carried by the bearing due to the combination of the induced axial load from the other bearing and the external axial load. However, only one of the bearings will carry the net axial load, and which one it is depends on the direction the bearings are mounted, the relative magnitudes of the induced loads, the direction of the external load, and whether the shaft or the housing is the moving part. Timken handles it with a table containing each of the configurations and a sign convention on the external loads. It further requires the application to be oriented horizontally with left and right bearings that must match the left and right sign conventions. Here, we will present a method that gives equivalent results, but that is perhaps more conducive to visualizing and understanding the logic behind it.

First, determine visually which bearing is being “squeezed” by the external thrust load, and label it as bearing A. Label the other bearing as bearing B. For example, in Fig. 11–16, the external thrust F_{ae} causes the shaft to push to the left against the cone of the left bearing, squeezing it against the rollers and the cup. On the other hand, it tends to pull apart the cup from the right bearing. The left bearing is therefore labeled as bearing A. If the direction of F_{ae} were reversed, then the right bearing would be labeled as bearing A. This approach to labeling the bearing being squeezed by the external thrust is applied similarly regardless of whether the bearings are mounted

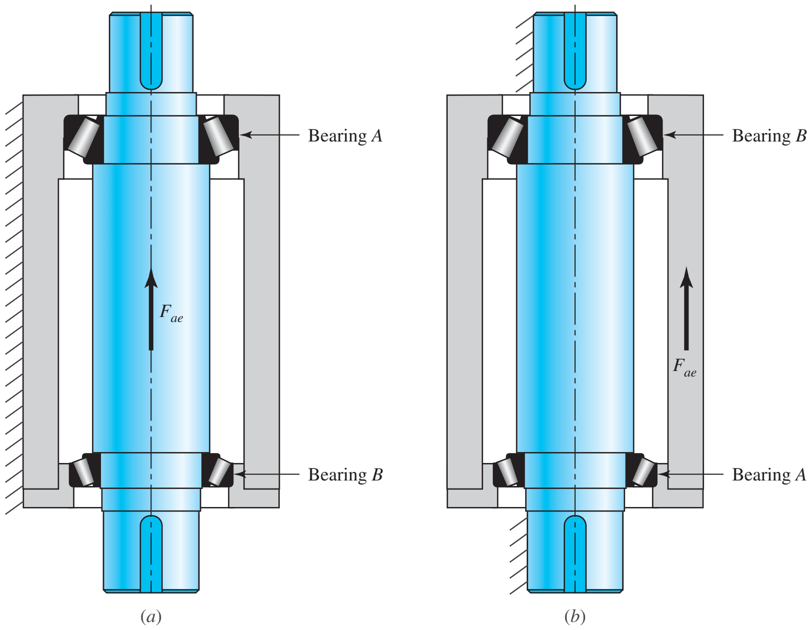


Figure 11–17

Examples of determining which bearing carries the external thrust load. In each case, the compressed bearing is labeled as bearing A. (a) External thrust applied to rotating shaft; (b) External thrust applied to rotating cylinder.

directly or indirectly, regardless of whether the shaft or the housing carries the external thrust, and regardless of the orientation of the assembly. To clarify by example, consider the vertical shaft and cylinder in Fig. 11–17 with direct-mounted bearings. In Fig. 11–17a, an external load is applied in the upward direction to a rotating shaft, compressing the top bearing, which should be labeled as bearing A. On the other hand, in Fig. 11–17b, an upward external load is applied to a rotating outer cylinder with a stationary shaft. In this case, the lower bearing is being squeezed and should be labeled as bearing A. If there is no external thrust, then either bearing can arbitrarily be labeled as bearing A.

Second, determine which bearing actually carries the net axial load. Generally, it would be expected that bearing A would carry the axial load, since the external thrust F_{ae} is directed toward A, along with the induced thrust F_{iB} from bearing B. However, if the induced thrust F_{iA} from bearing A happens to be larger than the combination of the external thrust and the thrust induced by bearing B, then bearing B will carry the net thrust load. We will use Eq. (a) for the bearing carrying the thrust load. Timken recommends leaving the other bearing at its original radial load, rather than reducing it due to the negative net thrust load. The results are presented in equation form below, where the induced thrusts are defined by Eq. (11–18).

$$\text{If} \quad F_{iA} \le (F_{iB} + F_{ae}) \quad \begin{cases} F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) & (11\text{–}19a) \\ F_{eB} = F_{rB} & (11\text{–}19b) \end{cases}$$

$$\text{If} \quad F_{iA} > (F_{iB} + F_{ae}) \quad \begin{cases} F_{eB} = 0.4F_{rB} + K_B(F_{iA} - F_{ae}) & (11\text{–}20a) \\ F_{eA} = F_{rA} & (11\text{–}20b) \end{cases}$$

In any case, if the equivalent radial load is ever less than the original radial load, then the original radial load should be used.

Once the equivalent radial loads are determined, they should be used to find the catalog rating load using any of Eqs. (11–3), (11–9), or (11–10) as before. Timken uses a Weibell model with $x_0 = 0$, $\theta = 4.48$, and $b = 3/2$. Note that since K_A and K_B are dependent on the specific bearing chosen, it may be necessary to iterate the process.

EXAMPLE 11–8

The shaft depicted in Fig. 11–18*a* carries a helical gear with a tangential force of 3980 N, a radial force of 1770 N, and a thrust force of 1690 N at the pitch cylinder with directions shown. The pitch diameter of the gear is 200 mm. The shaft runs at a speed of 800 rev/min, and the span (effective spread) between the direct-mount bearings is 150 mm. The design life is to be 5000 h and an application factor of 1 is appropriate. If the reliability of the bearing set is to be 0.99, select suitable single-row tapered-roller Timken bearings.

Solution

The reactions in the *xz* plane from Fig. 11–18*b* are

$$R_{zA} = \frac{3980(50)}{150} = 1327\text{ N}$$

$$R_{zB} = \frac{3980(100)}{150} = 2653\text{ N}$$

The reactions in the *xy* plane from Fig. 11–18*c* are

$$R_{yA} = \frac{1770(50)}{150} + \frac{169\,000}{150} = 1716.7 = 1717\text{ N}$$

$$R_{yB} = \frac{1770(100)}{150} - \frac{169\,000}{150} = 53.3\text{ N}$$

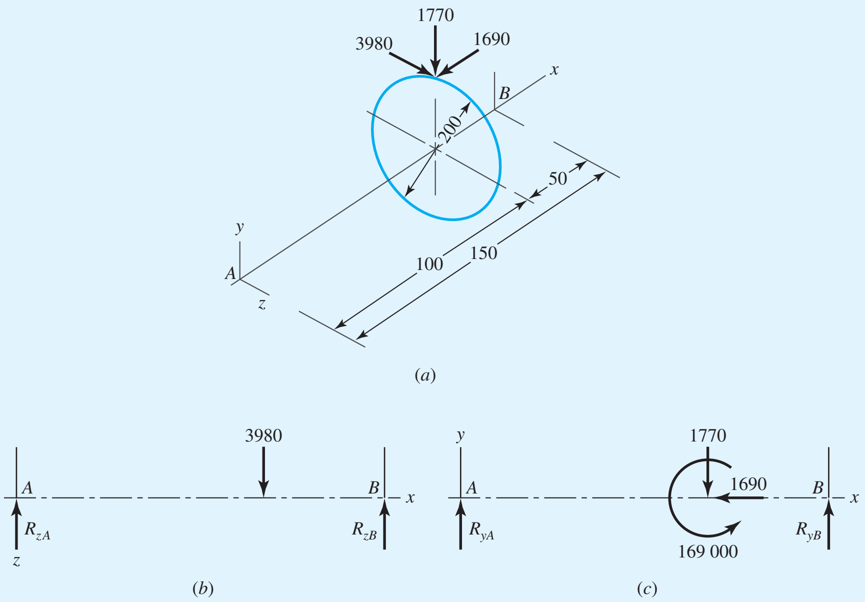


Figure 11–18

Essential geometry of helical gear and shaft. Length dimensions in mm, loads in N, couple in N · mm. (*a*) Sketch (not to scale) showing thrust, radial, and tangential forces. (*b*) Forces in *xz* plane. (*c*) Forces in *xy* plane.

The radial loads F_{rA} and F_{rB} are the vector additions of R_{yA} and R_{zA}, and R_{yB} and R_{zB}, respectively:

$$F_{rA} = (R_{zA}^2 + R_{yA}^2)^{1/2} = (1327^2 + 1717^2)^{1/2} = 2170 \text{ N}$$

$$F_{rB} = (R_{zB}^2 + R_{yB}^2)^{1/2} = (2653^2 + 53.3^2)^{1/2} = 2654 \text{ N}$$

Trial 1: With direct mounting of the bearings and application of the external thrust to the shaft, the squeezed bearing is bearing A as labeled in Fig. 11–18*a*. Using K of 1.5 as the initial guess for each bearing, the induced loads from the bearings are

$$F_{iA} = \frac{0.47F_{rA}}{K_A} = \frac{0.47(2170)}{1.5} = 680 \text{ N}$$

$$F_{iB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(2654)}{1.5} = 832 \text{ N}$$

Since F_{iA} is clearly less than $F_{iB} + F_{ae}$, bearing A carries the net thrust load, and Eq. (11–19) is applicable. Therefore, the dynamic equivalent loads are

$$F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) = 0.4(2170) + 1.5(832 + 1690) = 4651 \text{ N}$$

$$F_{eB} = F_{rB} = 2654 \text{ N}$$

The multiple of rating life is

$$x_D = \frac{L_D}{L_R} = \frac{\mathcal{L}_D n_D 60}{L_R} = \frac{(5000)(800)(60)}{90(10^6)} = 2.67$$

Estimate R_D as $\sqrt{0.99} = 0.995$ for each bearing. For bearing A, from Eq. (11–10) the catalog entry C_{10} should equal or exceed

$$C_{10} = (1)(4651)\left[\frac{2.67}{(4.48)(1 - 0.995)^{2/3}}\right]^{3/10} = 11\,486 \text{ N}$$

From Fig. 11–15, tentatively select type TS 15100 cone and 15245 cup, which will work: $K_A = 1.67$, $C_{10} = 12\,100$ N.

For bearing B, from Eq. (11–10), the catalog entry C_{10} should equal or exceed

$$C_{10} = (1)2654\left[\frac{2.67}{(4.48)(1 - 0.995)^{2/3}}\right]^{3/10} = 6554 \text{ N}$$

Tentatively select the bearing identical to bearing A, which will work: $K_B = 1.67$, $C_{10} = 12\,100$ N.

Trial 2: Repeat the process with $K_A = K_B = 1.67$ from tentative bearing selection.

$$F_{iA} = \frac{0.47F_{rA}}{K_A} = \frac{0.47(2170)}{1.67} = 611 \text{ N}$$

$$F_{iB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(2654)}{1.67} = 747 \text{ N}$$

Since F_{iA} is still less than $F_{iB} + F_{ae}$, Eq. (11–19) is still applicable.

$$F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) = 0.4(2170) + 1.67(747 + 1690) = 4938 \text{ N}$$

$$F_{eB} = F_{rB} = 2654 \text{ N}$$

For bearing A, from Eq. (11–10) the corrected catalog entry C_{10} should equal or exceed

$$C_{10} = (1)(4938)\left[\frac{2.67}{(4.48)(1 - 0.995)^{2/3}}\right]^{3/10} = 12\,195 \text{ N}$$

Although this catalog entry exceeds slightly the tentative selection for bearing A, we will keep it since the reliability of bearing B exceeds 0.995. In the next section we will quantitatively show that the combined reliability of bearing A and B will exceed the reliability goal of 0.99.

For bearing B, $F_{eB} = F_{rB} = 2654$ N. From Eq. (11–10),

$$C_{10} = (1)2654\left[\frac{2.67}{(4.48)(1 - 0.995)^{2/3}}\right]^{3/10} = 6554 \text{ N}$$

Select cone and cup 15100 and 15245, respectively, for both bearing A and B. Note from Fig. 11–14 the effective load center is located at $a = -5.8$ mm, that is, 5.8 mm into the cup from the back. Thus the shoulder-to-shoulder dimension should be $150 - 2(5.8) = 138.4$ mm. Note that in each iteration of Eq. (11–10) to find the catalog load rating, the bracketed portion of the equation is identical and need not be re-entered on a calculator each time.

11–10 Design Assessment for Selected Rolling-Contact Bearings

In textbooks, machine elements typically are treated singly. This can lead the reader to the presumption that a design assessment involves only that element, in this case a rolling-contact bearing. The immediately adjacent elements (the shaft journal and the housing bore) have immediate influence on the performance. Other elements, further removed (gears producing the bearing load), also have influence. Just as some say, “If you pull on something in the environment, you find that it is attached to everything else.” This should be intuitively obvious to those involved with machinery. How, then, can one check shaft attributes that aren’t mentioned in a problem statement? Possibly, because the bearing hasn’t been designed yet (in fine detail). All this points out the necessary iterative nature of designing, say, a speed reducer. If power, speed, and reduction are stipulated, then gear sets can be roughed in, their sizes, geometry, and location estimated, shaft forces and moments identified, bearings tentatively selected, seals identified; the bulk is beginning to make itself evident, the housing and lubricating scheme as well as the cooling considerations become clearer, shaft overhangs and coupling accommodations appear. It is time to iterate, now addressing each element again, knowing much more about all of the others. When you have completed the necessary iterations, you will know what you need for the design assessment for the bearings. In the meantime you do as much of the design assessment as you can, avoiding bad selections, even if tentative. Always keep in mind that you eventually have to do it all in order to pronounce your completed design satisfactory.

An outline of a design assessment for a rolling contact bearing includes, at a minimum,

- Bearing reliability for the load imposed and life expected
- Shouldering on shaft and housing satisfactory
- Journal finish, diameter and tolerance compatible
- Housing finish, diameter and tolerance compatible
- Lubricant type according to manufacturer's recommendations; lubricant paths and volume supplied to keep operating temperature satisfactory
- Preloads, if required, are supplied

Since we are focusing on rolling-contact bearings, we can address bearing reliability quantitatively, as well as shouldering. Other quantitative treatment will have to wait until the materials for shaft and housing, surface quality, and diameters and tolerances are known.

Bearing Reliability

Equation (11–9) can be solved for the reliability R_D in terms of C_{10}, the basic load rating of the selected bearing:

$$R = \exp\left(-\left\{\frac{x_D\left(\dfrac{a_f F_D}{C_{10}}\right)^a - x_0}{\theta - x_0}\right\}^b\right) \tag{11–21}$$

Equation (11–10) can likewise be solved for R_D:

$$R \approx 1 - \left\{\frac{x_D\left(\dfrac{a_f F_D}{C_{10}}\right)^a - x_0}{\theta - x_0}\right\}^b \qquad R \geq 0.90 \tag{11–22}$$

EXAMPLE 11–9

In Ex. 11–3, the minimum required load rating for 99 percent reliability, at $x_D = L_D/L_{10} = 540$, is $C_{10} = 6696$ lbf $= 29.8$ kN. From Table 11–2 a 02-40 mm deep-groove ball bearing would satisfy the requirement. If the bore in the application had to be 70 mm or larger (selecting a 02-70 mm deep-groove ball bearing), what is the resulting reliability?

Solution

From Table 11–2, for a 02-70 mm deep-groove ball bearing, $C_{10} = 61.8$ kN $=$ 13 888 lbf. Using Eq. (11–22), recalling from Ex. 11–3 that $a_f = 1.2$, $F_D = 413$ lbf, $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$, we can write

Answer

$$R \approx 1 - \left\{\frac{\left[540\left[\dfrac{1.2(413)}{13\,888}\right]^3 - 0.02\right]}{4.439}\right\}^{1.483} = 0.999\,963$$

which, as expected, is much higher than 0.99 from Ex. 11–3.

In tapered roller bearings, or other bearings for a two-parameter Weibull distribution, Eq. (11–21) becomes, for $x_0 = 0$, $\theta = 4.48$, $b = \frac{3}{2}$,

$$R = \exp\left\{-\left[\frac{x_D}{\theta[C_{10}/(a_f F_D)]^a}\right]^b\right\}$$

$$= \exp\left\{-\left[\frac{x_D}{4.48[C_{10}/(a_f F_D)]^{10/3}}\right]^{3/2}\right\} \qquad (11\text{–}23)$$

and Eq. (11–22) becomes

$$R \approx 1 - \left\{\frac{x_D}{\theta[C_{10}/(a_f F_D)]^a}\right\}^b = 1 - \left\{\frac{x_D}{4.48[C_{10}/(a_f F_D)]^{10/3}}\right\}^{3/2} \qquad (11\text{–}24)$$

EXAMPLE 11–10

In Ex. 11–8 bearings A and B (cone 15100 and cup 15245) have $C_{10} = 12\ 100$ N. What is the reliability of the pair of bearings A and B?

Solution

The desired life x_D was $5000(800)60/[90(10^6)] = 2.67$ rating lives. Using Eq. (11–24) for bearing A, where from Ex. 11–8, $F_D = F_{eA} = 4938$ N, and $a_f = 1$, gives

$$R_A \approx 1 - \left\{\frac{2.67}{4.48[12\ 100/(1 \times 4938)]^{10/3}}\right\}^{3/2} = 0.994\ 791$$

which is less than 0.995, as expected. Using Eq. (11–24) for bearing B with $F_D = F_{eB} = 2654$ N gives

$$R_B \approx 1 - \left\{\frac{2.67}{4.48[12\ 100/(1 \times 2654)]^{10/3}}\right\}^{3/2} = 0.999\ 766$$

Answer

The reliability of the bearing pair is

$$R = R_A R_B = 0.994\ 791(0.999\ 766) = 0.994\ 558$$

which is greater than the overall reliability goal of 0.99. When two bearings are made identical for simplicity, or reducing the number of spares, or other stipulation, and the loading is not the same, both can be made smaller and still meet a reliability goal. If the loading is disparate, then the more heavily loaded bearing can be chosen for a reliability goal just slightly larger than the overall goal.

An additional example is useful to show what happens in cases of pure thrust loading.

EXAMPLE 11–11 Consider a constrained housing as depicted in Fig. 11–19 with two direct-mount tapered roller bearings resisting an external thrust F_{ae} of 8000 N. The shaft speed is 950 rev/min, the desired life is 10 000 h, the expected shaft diameter is approximately 1 in. The reliability goal is 0.95. The application factor is appropriately $a_f = 1$.
(a) Choose a suitable tapered roller bearing for A.
(b) Choose a suitable tapered roller bearing for B.
(c) Find the reliabilities R_A, R_B, and R.

Solution (a) By inspection, note that the left bearing carries the axial load and is properly labeled as bearing A. The bearing reactions at A are

$$F_{rA} = F_{rB} = 0$$

$$F_{aA} = F_{ae} = 8000 \text{ N}$$

Since bearing B is unloaded, we will start with $R = R_A = 0.95$.

With no radial loads, there are no induced thrust loads. Eq. (11–19) is applicable.

$$F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) = K_A F_{ae}$$

If we set $K_A = 1$, we can find C_{10} in the thrust column and avoid iteration:

$$F_{eA} = (1)8000 = 8000 \text{ N}$$

$$F_{eB} = F_{rB} = 0$$

The multiple of rating life is

$$x_D = \frac{L_D}{L_R} = \frac{\mathcal{L}_D n_D 60}{L_R} = \frac{(10\,000)(950)(60)}{90(10^6)} = 6.333$$

Then, from Eq. (11–10), for bearing A

$$C_{10} = a_f F_{eA}\left[\frac{x_D}{4.48(1 - R_D)^{2/3}}\right]^{3/10}$$

$$= (1)8000\left[\frac{6.33}{4.48(1 - 0.95)^{2/3}}\right]^{3/10} = 16\,159 \text{ N}$$

Answer Figure 11–15 presents one possibility in the 1-in bore (25.4-mm) size: cone, HM88630, cup HM88610 with a thrust rating $(C_{10})_a = 17\,200$ N.

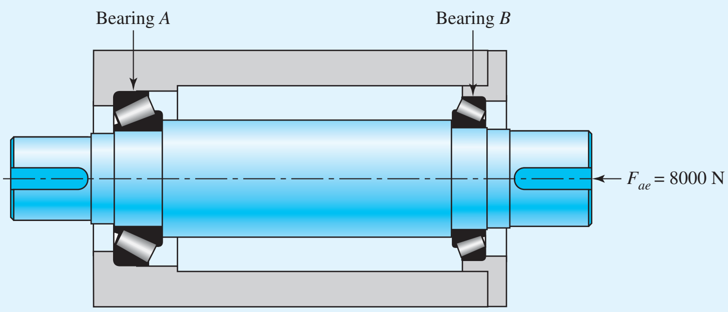


Figure 11–19

The constrained housing of Ex. 11–11.

Answer (*b*) Bearing *B* experiences no load, and the cheapest bearing of this bore size will do, including a ball or roller bearing.

(*c*) The actual reliability of bearing *A*, from Eq. (11–24), is

Answer
$$R_A \approx 1 - \left\{ \frac{x_D}{4.48[C_{10}/(a_f F_D)]^{10/3}} \right\}^{3/2}$$

$$\approx 1 - \left\{ \frac{6.333}{4.48[17\,200/(1 \times 8000)]^{10/3}} \right\}^{3/2} = 0.963$$

which is greater than 0.95, as one would expect. For bearing *B*,

Answer
$$F_D = F_{eB} = 0$$

$$R_B \approx 1 - \left[\frac{6.333}{0.85(17\,200/0)^{10/3}} \right]^{3/2} = 1 - 0 = 1$$

as one would expect. The combined reliability of bearings *A* and *B* as a pair is

Answer
$$R = R_A R_B = 0.963(1) = 0.963$$

which is greater than the reliability goal of 0.95, as one would expect.

Matters of Fit

Table 11–2 (and Fig. 11–8), which shows the rating of single-row, 02-series, deep-groove and angular-contact ball bearings, includes shoulder diameters recommended for the shaft seat of the inner ring and the shoulder diameter of the outer ring, denoted d_S and d_H, respectively. The shaft shoulder can be greater than d_S but not enough to obstruct the annulus. It is important to maintain concentricity and perpendicularity with the shaft centerline, and to that end the shoulder diameter should equal or exceed d_S. The housing shoulder diameter d_H is to be equal to or less than d_H to maintain concentricity and perpendicularity with the housing bore axis. Neither the shaft shoulder nor the housing shoulder features should allow interference with the free movement of lubricant through the bearing annulus.

In a tapered roller bearing (Fig. 11–15), the cup housing shoulder diameter should be equal to or less than D_b. The shaft shoulder for the cone should be equal to or greater than d_b. Additionally, free lubricant flow is not to be impeded by obstructing any of the annulus. In splash lubrication, common in speed reducers, the lubricant is thrown to the housing cover (ceiling) and is directed in its draining by ribs to a bearing. In direct mounting, a tapered roller bearing pumps oil from outboard to inboard. An oil passageway to the outboard side of the bearing needs to be provided. The oil returns to the sump as a consequence of bearing pump action. With an indirect mount, the oil is directed to the inboard annulus, the bearing pumping it to the outboard side. An oil passage from the outboard side to the sump has to be provided.

11–11 Lubrication

The contacting surfaces in rolling bearings have a relative motion that is both rolling and sliding, and so it is difficult to understand exactly what happens. If the relative velocity of the sliding surfaces is high enough, then the lubricant action is hydrodynamic (see Chap. 12). *Elastohydrodynamic lubrication* (EHD) is the phenomenon that occurs when a lubricant is introduced between surfaces that are in pure rolling contact. The contact of gear teeth and that found in rolling bearings and in cam-and-follower surfaces are typical examples. When a lubricant is trapped between two surfaces in rolling contact, a tremendous increase in the pressure within the lubricant film occurs. But viscosity is exponentially related to pressure, and so a very large increase in viscosity occurs in the lubricant that is trapped between the surfaces. Leibensperger[2] observes that the change in viscosity in and out of contact pressure is equivalent to the difference between cold asphalt and light sewing machine oil.

The purposes of an antifriction-bearing lubricant may be summarized as follows:

1. To provide a film of lubricant between the sliding and rolling surfaces
2. To help distribute and dissipate heat
3. To prevent corrosion of the bearing surfaces
4. To protect the parts from the entrance of foreign matter

Either oil or grease may be employed as a lubricant. The following rules may help in deciding between them.

Use Grease When	Use Oil When
1. The temperature is not over 200°F.	1. Speeds are high.
2. The speed is low.	2. Temperatures are high.
3. Unusual protection is required from the entrance of foreign matter.	3. Oiltight seals are readily employed.
4. Simple bearing enclosures are desired.	4. Bearing type is not suitable for grease lubrication.
5. Operation for long periods without attention is desired.	5. The bearing is lubricated from a central supply which is also used for other machine parts.

11–12 Mounting and Enclosure

There are so many methods of mounting antifriction bearings that each new design is a real challenge to the ingenuity of the designer. The housing bore and shaft outside diameter must be held to very close limits, which of course is expensive. There are usually one or more counterboring operations, several facing operations and drilling, tapping, and threading operations, all of which must be performed on the shaft, housing, or cover plate. Each of these operations contributes to the cost of production, so that the designer, in ferreting out a trouble-free and low-cost mounting, is faced with a difficult and important problem. The various bearing manufacturers' handbooks give many mounting details in almost every design area. In a text of this nature, however, it is possible to give only the barest details.

The most frequently encountered mounting problem is that which requires one bearing at each end of a shaft. Such a design might use one ball bearing at each end, one tapered roller bearing at each end, or a ball bearing at one end and a straight roller bearing at the other. One of the bearings usually has the added function of

[2] R. L. Leibensperger, "When Selecting a Bearing," *Machine Design,* vol. 47, no. 8, April 3, 1975, pp. 142–147.

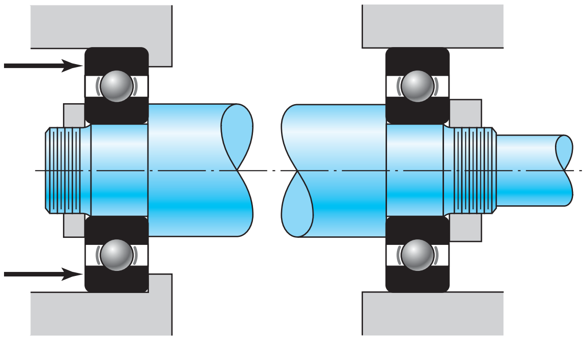


Figure 11–20

A common bearing mounting.

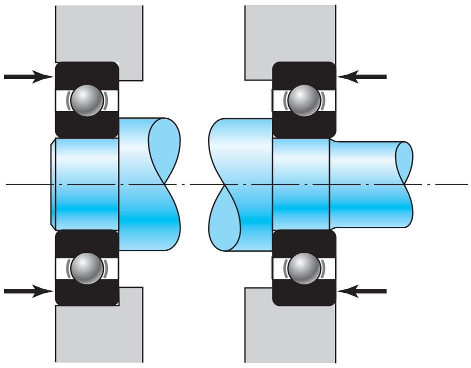


Figure 11–21

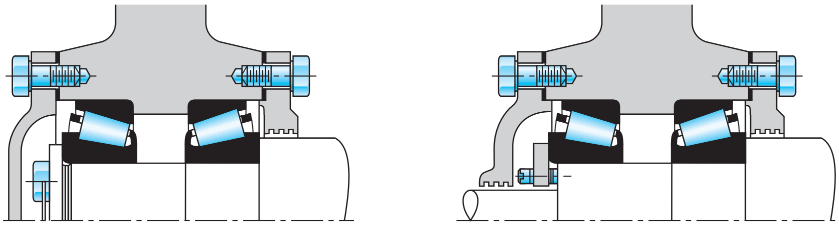
An alternative bearing mounting to that in Fig. 11–20.

positioning or axially locating the shaft. Figure 11–20 shows a very common solution to this problem. The inner rings are backed up against the shaft shoulders and are held in position by round nuts threaded onto the shaft. The outer ring of the left-hand bearing is backed up against a housing shoulder and is held in position by a device that is not shown. The outer ring of the right-hand bearing floats in the housing.

There are many variations possible on the method shown in Fig. 11–20. For example, the function of the shaft shoulder may be performed by retaining rings, by the hub of a gear or pulley, or by spacing tubes or rings. The round nuts may be replaced by retaining rings or by washers locked in position by screws, cotters, or taper pins. The housing shoulder may be replaced by a retaining ring; the outer ring of the bearing may be grooved for a retaining ring, or a flanged outer ring may be used. The force against the outer ring of the left-hand bearing is usually applied by the cover plate, but if no thrust is present, the ring may be held in place by retaining rings.

Figure 11–21 shows an alternative method of mounting in which the inner races are backed up against the shaft shoulders as before but no retaining devices are required. With this method the outer races are completely retained. This eliminates the grooves or threads, which cause stress concentration on the overhanging end, but it requires accurate dimensions in an axial direction or the employment of adjusting means. This method has the disadvantage that if the distance between the bearings is great, the temperature rise during operation may expand the shaft enough to destroy the bearings.

It is frequently necessary to use two or more bearings at one end of a shaft. For example, two bearings could be used to obtain additional rigidity or increased load capacity or to cantilever a shaft. Several two-bearing mountings are shown in Fig. 11–22. These may be used with tapered roller bearings, as shown, or with ball bearings. In either case it should be noted that the effect of the mounting is to preload the bearings in an axial direction.



(a) (b)

Figure 11–22

Two-bearing mountings. *(Courtesy of The Timken Company.)*

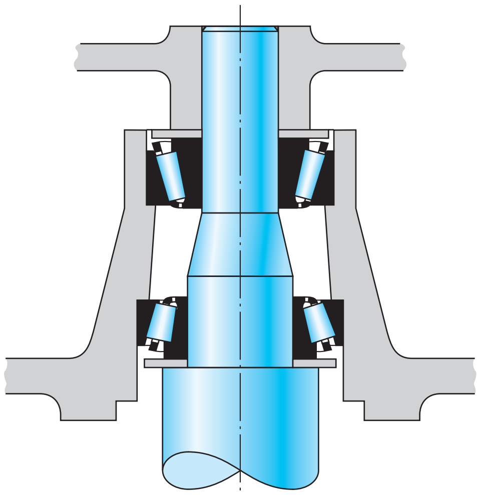
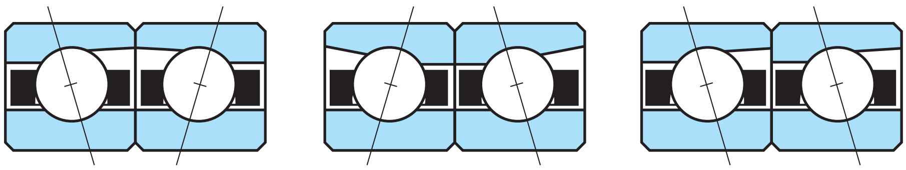


Figure 11–23

Mounting for a washing-machine spindle. *(Courtesy of The Timken Company.)*



(a) (b) (c)

Figure 11–24

Arrangements of angular ball bearings. (*a*) DF mounting; (*b*) DB mounting; (*c*) DT mounting. *(Courtesy of The Timken Company.)*

Figure 11–23 shows another two-bearing mounting. Note the use of washers against the cone backs.

When maximum stiffness and resistance to shaft misalignment is desired, pairs of angular-contact ball bearings (Fig. 11–2) are often used in an arrangement called *duplexing*. Bearings manufactured for duplex mounting have their rings ground with an offset, so that when a pair of bearings is tightly clamped together, a preload is automatically established. As shown in Fig. 11–24, three mounting arrangements are used. The face-to-face mounting, called DF, will take heavy radial loads and thrust loads from either direction. The DB mounting (back to back) has the greatest aligning stiffness and is also good for heavy radial loads and thrust loads from either direction. The tandem arrangement, called the DT mounting, is used where the thrust is always in the same direction; since the two bearings have their thrust functions in the same direction, a preload, if required, must be obtained in some other manner.

Bearings are usually mounted with the rotating ring a press fit, whether it be the inner or outer ring. The stationary ring is then mounted with a push fit. This permits the stationary ring to creep in its mounting slightly, bringing new portions of the ring into the load-bearing zone to equalize wear.

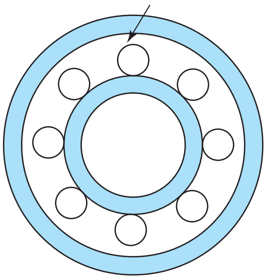


Figure 11–25

Clearance in an off-the-shelf bearing, exaggerated for clarity.

Preloading

The object of preloading is to remove the internal clearance usually found in bearings, to increase the fatigue life, and to decrease the shaft slope at the bearing. Figure 11–25 shows a typical bearing in which the clearance is exaggerated for clarity.

Preloading of straight roller bearings may be obtained by:

1. Mounting the bearing on a tapered shaft or sleeve to expand the inner ring
2. Using an interference fit for the outer ring
3. Purchasing a bearing with the outer ring preshrunk over the rollers

Ball bearings are usually preloaded by the axial load built in during assembly. However, the bearings of Fig. 11–24*a* and *b* are preloaded in assembly because of the differences in widths of the inner and outer rings.

It is always good practice to follow manufacturers' recommendations in determining preload, since too much will lead to early failure.

Alignment

The permissible misalignment in bearings depends on the type of bearing and the geometric and material properties of the specific bearing. Manufacturers' catalogs should be referenced for detailed specifications on a given bearing. In general, cylindrical and tapered roller bearings require alignments that are closer than deep-groove ball bearings. Spherical ball bearings and self-aligning bearings are the most forgiving. Table 7–2, p. 371, gives typical maximum ranges for each type of bearing. The life of the bearing decreases significantly when the misalignment exceeds the allowable limits.

Additional protection against misalignment is obtained by providing the full shoulders (see Fig. 11–8) recommended by the manufacturer. Also, if there is any misalignment at all, it is good practice to provide a safety factor of around 2 to account for possible increases during assembly.

Enclosures

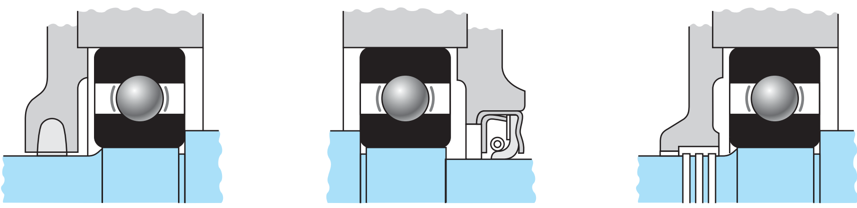
To exclude dirt and foreign matter and to retain the lubricant, the bearing mountings must include a seal. The three principal methods of sealings are the felt seal, the commercial seal, and the labyrinth seal (Fig. 11–26).

Felt seals may be used with grease lubrication when the speeds are low. The rubbing surfaces should have a high polish. Felt seals should be protected from dirt by placing them in machined grooves or by using metal stampings as shields.

The *commercial seal* is an assembly consisting of the rubbing element and, generally, a spring backing, which are retained in a sheet-metal jacket. These seals are usually made by press fitting them into a counterbored hole in the bearing cover. Since they obtain the sealing action by rubbing, they should not be used for high speeds.

Figure 11–26

Typical sealing methods. *(General Motors Corp. Used with permission, GM Media Archives.)*



(*a*) Felt seal (*b*) Commercial seal (*c*) Labyrinth seal

The *labyrinth seal* is especially effective for high-speed installations and may be used with either oil or grease. It is sometimes used with flingers. At least three grooves should be used, and they may be cut on either the bore or the outside diameter. The clearance may vary from 0.010 to 0.040 in, depending upon the speed and temperature.

PROBLEMS

Problems marked with an asterisk (*) are linked to problems in other chapters, as summarized in Table 1–2 of Sec. 1–17, p. 34.

Since each bearing manufacturer makes individual decisions with respect to materials, treatments, and manufacturing processes, manufacturers' experiences with bearing life distribution differ. In solving the following problems, we will use the experience of two manufacturers, tabulated in Table 11–6.

Table 11–6

Typical Weibull Parameters for Two Manufacturers

Manufacturer	Rating Life, Revolutions	Weibull Parameters Rating Lives		
		x_0	θ	b
1	$90(10^6)$	0	4.48	1.5
2	$1(10^6)$	0.02	4.459	1.483

Tables 11–2 and 11–3 are based on manufacturer 2.

11–1 A certain application requires a ball bearing with the inner ring rotating, with a design life of 25 kh at a speed of 350 rev/min. The radial load is 2.5 kN and an application factor of 1.2 is appropriate. The reliability goal is 0.90. Find the multiple of rating life required, x_D, and the catalog rating C_{10} with which to enter a bearing table. Choose a 02-series deep-groove ball bearing from Table 11–2, and estimate the reliability in use.

11–2 An angular-contact, inner ring rotating, 02-series ball bearing is required for an application in which the life requirement is 40 kh at 520 rev/min. The design radial load is 725 lbf. The application factor is 1.4. The reliability goal is 0.90. Find the multiple of rating life x_D required and the catalog rating C_{10} with which to enter Table 11–2. Choose a bearing and estimate the existing reliability in service.

11–3 The other bearing on the shaft of Prob. 11–2 is to be a 03-series cylindrical roller bearing with inner ring rotating. For a 2235-lbf radial load, find the catalog rating C_{10} with which to enter Table 11–3. The reliability goal is 0.90. Choose a bearing and estimate its reliability in use.

11–4 Problems 11–2 and 11–3 raise the question of the reliability of the bearing pair on the shaft. Since the combined reliabilities R is R_1R_2, what is the reliability of the two bearings (probability that either or both will not fail) as a result of your decisions in Probs. 11–2 and 11–3? What does this mean in setting reliability goals for each of the bearings of the pair on the shaft?

11–5 Combine Probs. 11–2 and 11–3 for an overall reliability of $R = 0.90$. Reconsider your selections, and meet this overall reliability goal.

11–6 A straight (cylindrical) roller bearing is subjected to a radial load of 20 kN. The life is to be 8000 h at a speed of 950 rev/min and exhibit a reliability of 0.95. What basic load rating should be used in selecting the bearing from a catalog of manufacturer 2 in Table 11–6?

11–7 Two ball bearings from different manufacturers are being considered for a certain application. Bearing A has a catalog rating of 2.0 kN based on a catalog rating system of 3 000 hours at

500 rev/min. Bearing *B* has a catalog rating of 7.0 kN based on a catalog that rates at 10^6 cycles. For a given application, determine which bearing can carry the larger load.

11–8 to 11–13 For the bearing application specifications given in the table for the assigned problem, determine the Basic Load Rating for a ball bearing with which to enter a bearing catalog of manufacturer 2 in Table 11–6. Assume an application factor of one.

Problem Number	Radial Load	Design Life	Desired Reliability
11–8	2 kN	10^9 rev	90%
11–9	800 lbf	12 kh, 350 rev/min	90%
11–10	4 kN	8 kh, 500 rev/min	90%
11–11	650 lbf	5 yrs, 40 h/week, 400 rev/min	95%
11–12	9 kN	10^8 rev	99%
11–13	11 kips	20 kh, 200 rev/min	99%

11–14* to 11–17* For the problem specified in the table, build upon the results of the original problem to obtain a Basic Load Rating for a ball bearing at *C* with a 95 percent reliability, assuming distribution data from manufacturer 2 in Table 11–6. The shaft rotates at 1200 rev/min, and the desired bearing life is 15 kh. Use an application factor of 1.2.

Problem Number	Original Problem, Page Number
11–14*	3–68, 151
11–15*	3–69, 151
11–16*	3–70, 151
11–17*	3–71, 151

11–18* For the shaft application defined in Prob. 3–77, p. 153, the input shaft *EG* is driven at a constant speed of 191 rev/min. Obtain a Basic Load Rating for a ball bearing at *A* for a life of 12 kh with a 95 percent reliability, assuming distribution data from manufacturer 2 in Table 11–6.

11–19* For the shaft application defined in Prob. 3–79, p. 153, the input shaft *EG* is driven at a constant speed of 280 rev/min. Obtain a Basic Load Rating for a cylindrical roller bearing at *A* for a life of 14 kh with a 98 percent reliability, assuming distribution data from manufacturer 2 in Table 11–6.

11–20 An 02-series single-row deep-groove ball bearing with a 65-mm bore (see Tables 11–1 and 11–2 for specifications) is loaded with a 3-kN axial load and a 7-kN radial load. The outer ring rotates at 500 rev/min.

(*a*) Determine the equivalent radial load that will be experienced by this particular bearing.

(*b*) Determine whether this bearing should be expected to carry this load with a 95 percent reliability for 10 kh.

11–21 An 02-series single-row deep-groove ball bearing with a 30-mm bore (see Tables 11–1 and 11–2 for specifications) is loaded with a 2-kN axial load and a 5-kN radial load. The inner ring rotates at 400 rev/min.

(*a*) Determine the equivalent radial load that will be experienced by this particular bearing.

(*b*) Determine the predicted life (in revolutions) that this bearing could be expected to give in this application with a 99 percent reliability.

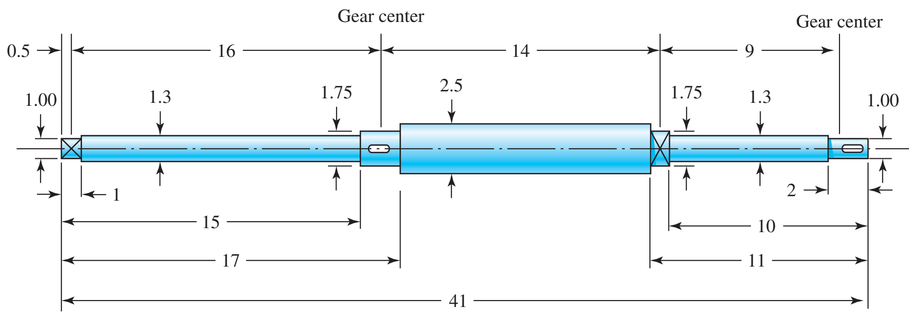
11–22 to 11–26 An 02-series single-row deep-groove ball bearing is to be selected from Table 11–2 for the application conditions specified in the table. Assume Table 11–1 is applicable if needed. Specify the smallest bore size from Table 11–2 that can satisfy these conditions.

Problem Number	Radial Load	Axial Load	Design Life	Ring Rotating	Desired Reliability
11–22	8 kN	0 kN	10^9 rev	Inner	90%
11–23	8 kN	2 kN	10 kh, 400 rev/min	Inner	99%
11–24	8 kN	3 kN	10^8 rev	Outer	90%
11–25	10 kN	5 kN	12 kh, 300 rev/min	Inner	95%
11–26	9 kN	3 kN	10^8 rev	Outer	99%

11–27* The shaft shown in the figure is proposed as a preliminary design for the application defined in Prob. 3–72, p. 152. The effective centers of the gears for force transmission are shown. The dimensions for the bearing surfaces (indicated with cross markings) have been estimated. The shaft rotates at 1200 rev/min, and the desired bearing life is 15 kh with a 95 percent reliability in each bearing, assuming distribution data from manufacturer 2 in Table 11–6. Use an application factor of 1.2.

(*a*) Obtain a Basic Load Rating for a ball bearing at the right end.

(*b*) Use an online bearing catalog to find a specific bearing that satisfies the needed Basic Load Rating and the geometry requirements. If necessary, indicate appropriate adjustments to the dimensions of the bearing surface.



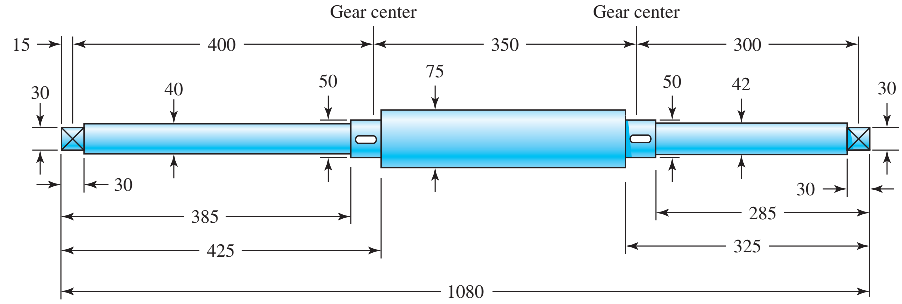
*Problem 11–27**
All fillets $\frac{1}{16}$ in. Dimensions in inches.

11–28* Repeat the requirements of Prob. 11–27 for the bearing at the left end of the shaft.

11–29* The shaft shown in the figure is proposed as a preliminary design for the application defined in Prob. 3–73, p. 152. The effective centers of the gears for force transmission are shown. The dimensions for the bearing surfaces (indicated with cross markings) have been estimated. The shaft rotates at 900 rev/min, and the desired bearing life is 12 kh with a 98 percent reliability in each bearing, assuming distribution data from manufacturer 2 in Table 11–6. Use an application factor of 1.2.

(*a*) Obtain a Basic Load Rating for a ball bearing at the right end.

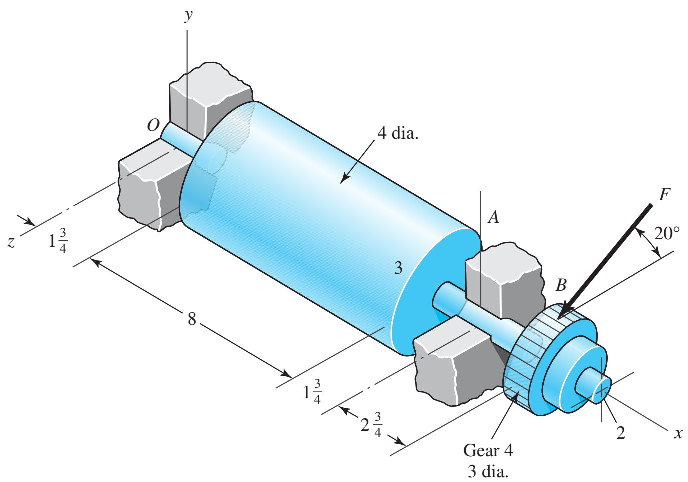
(*b*) Use an online bearing catalog to find a specific bearing that satisfies the needed Basic Load Rating and the geometry requirements. If necessary, indicate appropriate adjustments to the dimensions of the bearing surface.



*Problem 11–29**
All fillets 2 mm. Dimensions in millimeters.

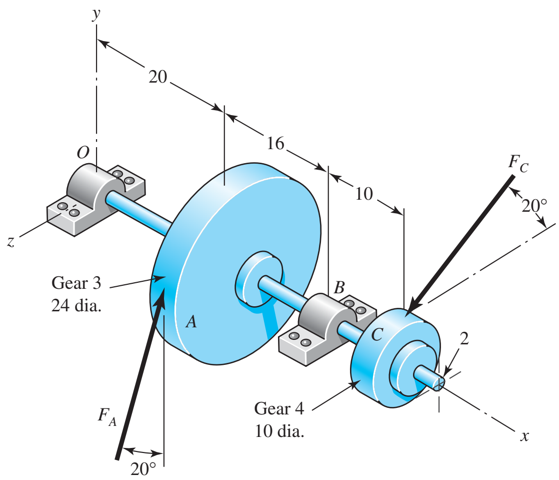
11–30* Repeat the requirements of Prob. 11–29 for the bearing at the left end of the shaft.

11–31 Shown in the figure is a gear-driven squeeze roll that mates with an idler roll. The roll is designed to exert a normal force of 35 lbf/in of roll length and a pull of 28 lbf/in on the material being processed. The roll speed is 350 rev/min, and a design life of 35 kh is desired. Use an application factor of 1.2, and select a pair of angular-contact 02-series ball bearings from Table 11–2 to be mounted at 0 and *A*. Use the same size bearings at both locations and a combined reliability of at least 0.92, assuming distribution data from manufacturer 2 in Table 11–6.



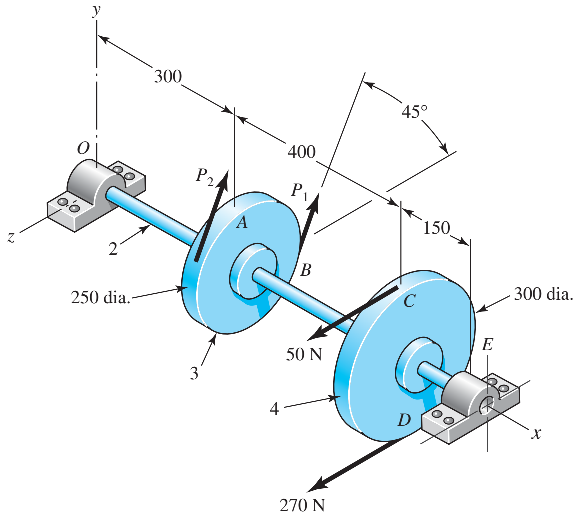
Problem 11–31
Dimensions in inches.

11–32 The figure shown is a geared countershaft with an overhanging pinion at *C*. Select an angular-contact ball bearing from Table 11–2 for mounting at *O* and an 02-series cylindrical roller bearing from Table 11–3 for mounting at *B*. The force on gear *A* is $F_A = 600$ lbf, and the shaft is to run at a speed of 420 rev/min. Solution of the statics problem gives force of bearings against the shaft at *O* as $\mathbf{R}_O = -387\mathbf{j} + 467\mathbf{k}$ lbf, and at *B* as $\mathbf{R}_B = 316\mathbf{j} - 1615\mathbf{k}$ lbf. Specify the bearings required, using an application factor of 1.2, a desired life of 40 kh, and a combined reliability goal of 0.95, assuming distribution data from manufacturer 2 in Table 11–6.



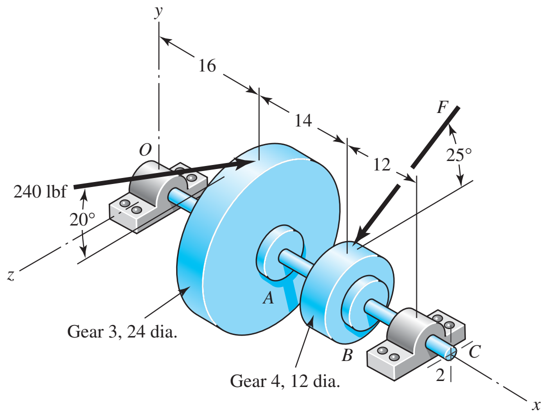
Problem 11–32
Dimensions in inches.

11–33 The figure is a schematic drawing of a countershaft that supports two V-belt pulleys. The countershaft runs at 1500 rev/min and the bearings are to have a life of 60 kh at a combined reliability of 0.98, assuming distribution data from manufacturer 2 in Table 11–6. The belt tension on the loose side of pulley *A* is 15 percent of the tension on the tight side. Select deep-groove bearings from Table 11–2 for use at *O* and *E*, using an application factor of unity.



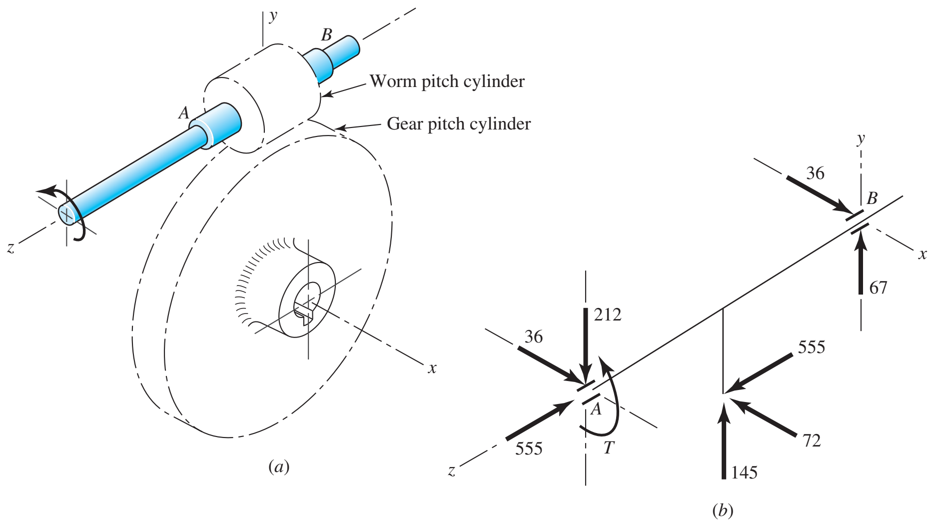
Problem 11–33
Dimensions in millimeters.

11–34 A gear-reduction unit uses the countershaft depicted in the figure. Find the two bearing reactions. The bearings are to be angular-contact ball bearings, having a desired life of 50 kh when used at 300 rev/min. Use 1.2 for the application factor and a reliability goal for the bearing pair of 0.96, assuming distribution data from manufacturer 2 in Table 11–6. Select the bearings from Table 11–2.



Problem 11–34
Dimensions in inches.

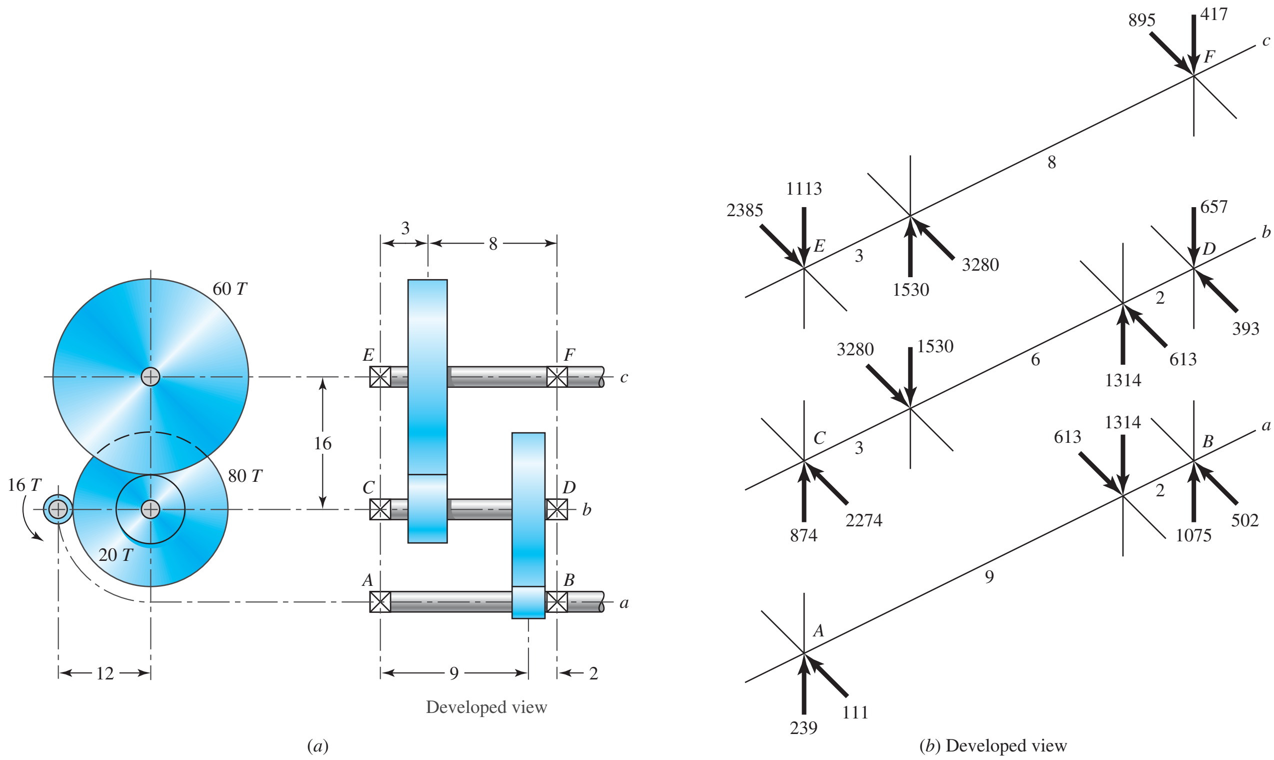
11–35 The worm shaft shown in part *a* of the figure transmits 1.2 hp at 500 rev/min. A static force analysis gave the results shown in part *b* of the figure. Bearing *A* is to be an angular-contact ball bearing selected from Table 11–2, mounted to take the 555-lbf thrust load. The bearing at *B* is to take only the radial load, so an 02-series cylindrical roller bearing from Table 11–3 will be employed. Use an application factor of 1.2, a desired life of 30 kh, and a combined reliability goal of 0.99, assuming distribution data from manufacturer 2 in Table 11–6. Specify each bearing.



Problem 11–35
(*a*) Worm and worm gear;
(*b*) force analysis of worm shaft, forces in pounds.

11–36 In bearings tested at 2000 rev/min with a steady radial load of 18 kN, a set of bearings showed an L_{10} life of 115 h and an L_{80} life of 600 h. The basic load rating of this bearing is 39.6 kN. Estimate the Weibull shape factor *b* and the characteristic life θ for a two-parameter model. This manufacturer rates ball bearings at 1 million revolutions.

11–37 A 16-tooth pinion drives the double-reduction spur-gear train in the figure. All gears have 25° pressure angles. The pinion rotates ccw at 1200 rev/min and transmits power to the gear train. The shaft has not yet been designed, but the free bodies have been generated. The shaft speeds are 1200 rev/min, 240 rev/min, and 80 rev/min. A bearing study is commencing with a 10-kh life and a gearbox bearing ensemble reliability of 0.99, assuming distribution data from manufacturer 2 in Table 11–6. An application factor of 1.2 is appropriate. For each shaft, specify a matched pair of 02-series cylindrical roller bearings from Table 11–3.



Problem 11–37

(*a*) Drive detail; (*b*) force analysis on shafts. Forces in pounds; linear dimensions in inches.

11–38 Estimate the remaining life in revolutions of an 02-30 mm angular-contact ball bearing already subjected to 200 000 revolutions with a radial load of 18 kN, if it is now to be subjected to a change in load to 30 kN.

11–39 The same 02-30 angular-contact ball bearing as in Prob. 11–38 is to be subjected to a two-step loading cycle of 4 min with a loading of 18 kN, and one of 6 min with a loading of 30 kN. This cycle is to be repeated until failure. Estimate the total life in revolutions, hours, and loading cycles.

11–40 A countershaft is supported by two tapered roller bearings using an indirect mounting. The radial bearing loads are 560 lbf for the left-hand bearing and 1095 for the right-hand bearing. An axial load of 200 lbf is carried by the left bearing. The shaft rotates at 400 rev/min and is to have a desired life of 40 kh. Use an application factor of 1.4 and a combined reliability goal of 0.90, assuming distribution data from manufacturer 1 in Table 11–6. Using an initial $K = 1.5$, find the required radial rating for each bearing. Select the bearings from Fig. 11–15.

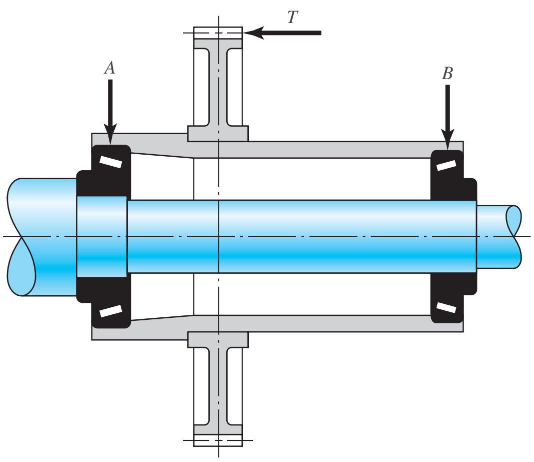
11–41* For the shaft application defined in Prob. 3–74, p. 152, perform a preliminary specification for tapered roller bearings at C and D. A bearing life of 10^8 revolutions is desired with a 90 percent combined reliability for the bearing set, assuming distribution data from manufacturer 1 in

Table 11–6. Should the bearings be oriented with direct mounting or indirect mounting for the axial thrust to be carried by the bearing at *C*? Assuming bearings are available with $K = 1.5$, find the required radial rating for each bearing. For this preliminary design, assume an application factor of one.

11–42* For the shaft application defined in Prob. 3–76, p. 153, perform a preliminary specification for tapered roller bearings at *A* and *B*. A bearing life of 500 million revolutions is desired with a 90 percent combined reliability for the bearing set, assuming distribution data from manufacturer 1 in Table 11–6. Should the bearings be oriented with direct mounting or indirect mounting for the axial thrust to be carried by the bearing at *A*? Assuming bearings are available with $K = 1.5$, find the required radial rating for each bearing. For this preliminary design, assume an application factor of one.

11–43 An outer hub rotates around a stationary shaft, supported by two tapered roller bearings as shown in Fig. 11–23. The device is to operate at 250 rev/min, 8 hours per day, 5 days per week, for 5 years, before bearing replacement is necessary. A reliability of 90 percent on each bearing is acceptable. A free body analysis determines the radial force carried by the upper bearing to be 12 kN and the radial force at the lower bearing to be 25 kN. In addition, the outer hub applies a downward force of 5 kN. Assuming bearings are available from manufacturer 1 in Table 11–6 with $K = 1.5$, find the required radial rating for each bearing. Assume an application factor of 1.2.

11–44 The gear-reduction unit shown has a gear that is press fit onto a cylindrical sleeve that rotates around a stationary shaft. The helical gear transmits an axial thrust load *T* of 250 lbf as shown in the figure. Tangential and radial loads (not shown) are also transmitted through the gear, producing radial ground reaction forces at the bearings of 875 lbf for bearing *A* and 625 lbf for bearing *B*. The desired life for each bearing is 90 kh at a speed of 150 rev/min with a 90 percent reliability. The first iteration of the shaft design indicates approximate diameters of $1\frac{1}{8}$ in at *A* and 1 in at *B*. Assuming distribution data from manufacturer 1 in Table 11–6, select suitable tapered roller bearings from Fig. 11–15.



Problem 11–44
(Courtesy of The Timken Company.)

12 Lubrication and Journal Bearings

Chapter Outline

The object of lubrication is to reduce friction, wear, and heating of machine parts that move relative to each other. A lubricant is any substance that, when inserted between the moving surfaces, accomplishes these purposes. In a sleeve bearing, a shaft, or *journal,* rotates or oscillates within a sleeve, or *bushing,* and the relative motion is sliding. In an antifriction bearing, the main relative motion is rolling. A follower may either roll or slide on the cam. Gear teeth mate with each other by a combination of rolling and sliding. Pistons slide within their cylinders. All these applications require lubrication to reduce friction, wear, and heating.

The field of application for journal bearings is immense. The crankshaft and connecting-rod bearings of an automotive engine must operate for thousands of miles at high temperatures and under varying load conditions. The journal bearings used in the steam turbines of power-generating stations are said to have reliabilities approaching 100 percent. At the other extreme there are thousands of applications in which the loads are light and the service relatively unimportant; a simple, easily installed bearing is required, using little or no lubrication. In such cases an antifriction bearing might be a poor answer because of the cost, the elaborate enclosures, the close tolerances, the radial space required, the high speeds, or the increased inertial effects. Instead, a nylon bearing requiring no lubrication, a powder-metallurgy bearing with the lubrication "built in," or a bronze bearing with ring oiling, wick feeding, or solid-lubricant film or grease lubrication might be a very satisfactory solution. Recent metallurgy developments in bearing materials, combined with increased knowledge of the lubrication process, now make it possible to design journal bearings with satisfactory lives and very good reliabilities.

Much of the material we have studied thus far in this book has been based on fundamental engineering studies, such as statics, dynamics, the mechanics of solids, metal processing, mathematics, and metallurgy. In the study of lubrication and journal bearings, additional fundamental studies, such as chemistry, fluid mechanics, thermodynamics, and heat transfer, must be utilized in developing the material. While we shall not utilize all of them in the material to be included here, you can now begin to appreciate better how the study of mechanical engineering design is really an integration of most of your previous studies and a directing of this total background toward the resolution of a single objective.

12–1 Types of Lubrication

Five distinct forms of lubrication may be identified:

1. Hydrodynamic
2. Hydrostatic
3. Elastohydrodynamic
4. Boundary
5. Solid film

Hydrodynamic lubrication means that the load-carrying surfaces of the bearing are separated by a relatively thick film of lubricant, so as to prevent metal-to-metal contact, and that the stability thus obtained can be explained by the laws of fluid mechanics. Hydrodynamic lubrication does not depend upon the introduction of the lubricant under pressure, though that may occur; but it does require the existence of an adequate supply at all times. The film pressure is created by the moving surface itself pulling the lubricant into a wedge-shaped zone at a velocity sufficiently high to create the pressure necessary to separate the surfaces against the load on the bearing. Hydrodynamic lubrication is also called *full-film,* or *fluid, lubrication*.

Hydrostatic lubrication is obtained by introducing the lubricant, which is sometimes air or water, into the load-bearing area at a pressure high enough to separate the surfaces with a relatively thick film of lubricant. So, unlike hydrodynamic lubrication, this kind of lubrication does not require motion of one surface relative to another. We shall not deal with hydrostatic lubrication in this book, but the subject should be considered in designing bearings where the velocities are small or zero and where the frictional resistance is to be an absolute minimum.

Elastohydrodynamic lubrication is the phenomenon that occurs when a lubricant is introduced between surfaces that are in rolling contact, such as mating gears or rolling bearings. The mathematical explanation requires the Hertzian theory of contact stress and fluid mechanics.

Insufficient surface area, a drop in the velocity of the moving surface, a lessening in the quantity of lubricant delivered to a bearing, an increase in the bearing load, or an increase in lubricant temperature resulting in a decrease in viscosity—any one of these—may prevent the buildup of a film thick enough for full-film lubrication. When this happens, the highest asperities may be separated by lubricant films only several molecular dimensions in thickness. This is called *boundary lubrication*. The change from hydrodynamic to boundary lubrication is not at all a sudden or abrupt one. It is probable that a mixed hydrodynamic- and boundary-type lubrication occurs first, and as the surfaces move closer together, the boundary-type lubrication becomes predominant. The viscosity of the lubricant is not of as much importance with boundary lubrication as is the chemical composition.

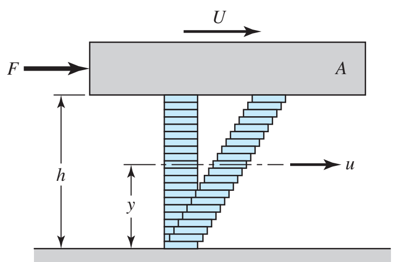
When bearings must be operated at extreme temperatures, a *solid-film lubricant* such as graphite or molybdenum disulfide must be used because the ordinary mineral oils are not satisfactory. Much research is currently being carried out in an effort to find composite bearing materials with low wear rates as well as small frictional coefficients.

12–2 Viscosity

In Fig. 12–1 let plate A be moving with a velocity U on a film of lubricant of thickness h. We imagine the film as composed of a series of horizontal layers and the force F causing these layers to deform or slide on one another just like a deck of cards. The layers in contact with the moving plate are assumed to have a velocity U; those in contact with the stationary surface are assumed to have a zero velocity. Intermediate layers have velocities that depend upon their distances y from the stationary surface. Newton's viscous effect states that the shear stress in the fluid is proportional to the rate of change of velocity with respect to y. Thus

$$\tau = \frac{F}{A} = \mu \frac{du}{dy} \tag{12–1}$$

Figure 12–1



where μ is the constant of proportionality and defines *absolute viscosity,* also called *dynamic viscosity*. The derivative du/dy is the rate of change of velocity with distance and may be called the rate of shear, or the velocity gradient. The viscosity μ is thus a measure of the internal frictional resistance of the fluid. For most lubricating fluids, the rate of shear is constant, and $du/dy = U/h$. Thus, from Eq. (12–1),

$$\tau = \frac{F}{A} = \mu \frac{U}{h} \tag{12–2}$$

Fluids exhibiting this characteristic are said to be *Newtonian fluids*. The unit of viscosity in the ips system is seen to be the pound-force-second per square inch; this is the same as stress or pressure multiplied by time. The ips unit is called the *reyn,* in honor of Sir Osborne Reynolds.

The absolute viscosity is measured by the pascal-second (Pa · s) in SI; this is the same as a Newton-second per square meter. The conversion from ips units to SI is the same as for stress. For example, multiply the absolute viscosity in reyns by 6890 to convert to units of Pa · s.

The American Society of Mechanical Engineers (ASME) has published a list of cgs units that are not to be used in ASME documents.[1] This list results from a recommendation by the International Committee of Weights and Measures (CIPM) that the use of cgs units with special names be discouraged. Included in this list is a unit of force called the *dyne* (dyn), a unit of dynamic viscosity called the *poise* (P), and a unit of kinematic viscosity called the *stoke* (St). All of these units have been, and still are, used extensively in lubrication studies.

The poise is the cgs unit of dynamic or absolute viscosity, and its unit is the dyne-second per square centimeter (dyn · s/cm^2). It has been customary to use the centipoise (cP) in analysis, because its value is more convenient. When the viscosity is expressed in centipoises, it is designated by Z. The conversion from cgs units to SI and ips units is as follows:

$$\mu(\text{Pa} \cdot \text{s}) = (10)^{-3} Z\,(\text{cP})$$

$$\mu(\text{reyn}) = \frac{Z\,(\text{cP})}{6.89(10)^6}$$

$$\mu(\text{mPa} \cdot \text{s}) = 6.89\,\mu'\,(\mu\text{reyn})$$

In using ips units, the microreyn (μreyn) is often more convenient. The symbol μ' will be used to designate viscosity in μreyn such that $\mu = \mu'/(10^6)$.

The ASTM standard method for determining viscosity uses an instrument called the Saybolt Universal Viscosimeter. The method consists of measuring the time in seconds for 60 mL of lubricant at a specified temperature to run through a tube 17.6 mm in diameter and 12.25 mm long. The result is called the *kinematic viscosity,* and in the past the unit of the square centimeter per second has been used. One square centimeter per second is defined as a *stoke*. By the use of the *Hagen-Poiseuille law,* the kinematic viscosity based upon seconds Saybolt, also called *Saybolt Universal viscosity* (SUV) in seconds, is

$$Z_k = \left(0.22t - \frac{180}{t}\right) \tag{12–3}$$

where Z_k is in centistokes (cSt) and t is the number of seconds Saybolt.

[1] *ASME Orientation and Guide for Use of Metric Units,* 2nd ed., American Society of Mechanical Engineers, 1972, p. 13.

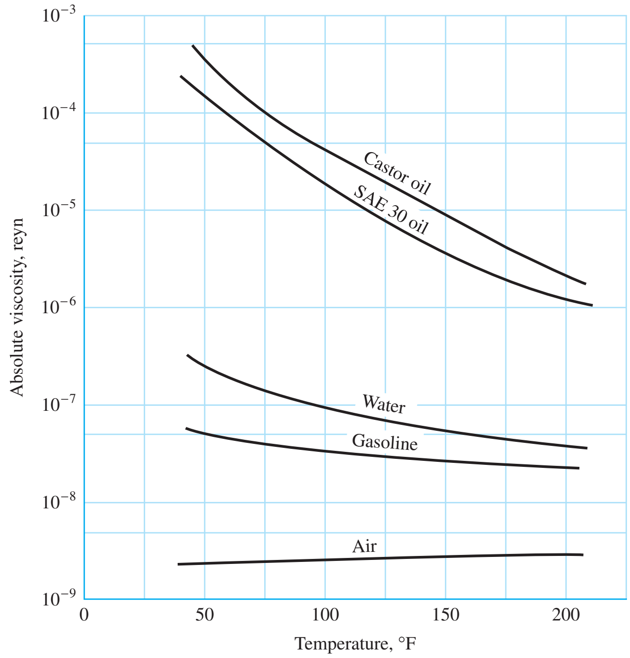


Figure 12–2

A comparison of the viscosities of various fluids.

In SI, the kinematic viscosity ν has the unit of the square meter per second (m^2/s), and the conversion is

$$\nu(m^2/s) = 10^{-6} Z_k \text{ (cSt)}$$

Thus, Eq. (12–3) becomes

$$\nu = \left(0.22t - \frac{180}{t}\right)(10^{-6}) \tag{12–4}$$

To convert to dynamic viscosity, we multiply ν by the density in SI units. Designating the density as ρ with the unit of the kilogram per cubic meter, we have

$$\mu = \rho\left(0.22t - \frac{180}{t}\right)(10^{-6}) \tag{12–5}$$

where μ is in pascal-seconds.

Figure 12–2 shows the absolute viscosity in the ips system of a number of fluids often used for lubrication purposes and their variation with temperature.

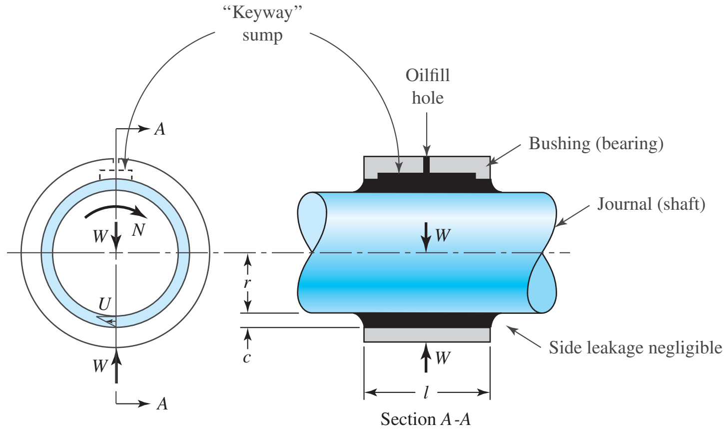
12–3 Petroff's Equation

The phenomenon of bearing friction was first explained by Petroff on the assumption that the shaft is concentric with its bushing. Though we shall seldom make use of Petroff's method of analysis in the material to follow, it is important because it defines groups of dimensionless parameters and because the coefficient of friction predicted by this law turns out to be quite good even when the shaft is not concentric.

Let us now consider a vertical shaft rotating in a guide bearing. It is assumed that the bearing carries a very small load, that the clearance space is completely filled with oil, and that leakage is negligible (Fig. 12–3). We denote the radius of the shaft

Figure 12–3

Petroff's lightly loaded journal bearing consisting of a shaft journal and a bushing with an axial-groove internal lubricant reservoir. The linear velocity gradient is shown in the end view. The clearance c is several thousandths of an inch and is grossly exaggerated for presentation purposes.



by r, the radial clearance by c, and the length of the bearing by l, all dimensions being in inches. If the shaft rotates at N rev/s, then its surface velocity is $U = 2\pi r N$ in/s. Since the shearing stress in the lubricant is equal to the velocity gradient times the viscosity, from Eq. (12–2) we have

$$\tau = \mu \frac{U}{h} = \frac{2\pi r \mu N}{c} \tag{a}$$

where the radial clearance c has been substituted for the distance h. The force required to shear the film is the stress times the area. The torque is the force times the lever arm r. Thus

$$T = (\tau A)(r) = \left(\frac{2\pi r \mu N}{c}\right)(2\pi r l)(r) = \frac{4\pi^2 r^3 l \mu N}{c} \tag{b}$$

If we now designate a small force on the bearing by W, in pounds-force, then the pressure P, in pounds-force per square inch of projected area, is $P = W/2rl$. The frictional force is fW, where f is the coefficient of friction, and so the frictional torque is

$$T = fWr = (f)(2rlP)(r) = 2r^2 flP \tag{c}$$

Substituting the value of the torque from Eq. (c) in Eq. (b) and solving for the coefficient of friction, we find

$$f = 2\pi^2 \frac{\mu N}{P} \frac{r}{c} \tag{12–6}$$

Equation (12–6) is called *Petroff's equation* and was first published in 1883. The two quantities $\mu N/P$ and r/c are very important parameters in lubrication. Substitution of the appropriate dimensions in each parameter will show that they are dimensionless.

The *bearing characteristic number,* or the *Sommerfeld number,* is defined by the equation

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} \tag{12–7}$$

The Sommerfeld number is very important in lubrication analysis because it contains many of the parameters that are specified by the designer. Note that it is also dimensionless. The quantity r/c is called the *radial clearance ratio*. If we multiply both sides of Eq. (12–6) by this ratio, we obtain the interesting relation

$$f\frac{r}{c} = 2\pi^2 \frac{\mu N}{P}\left(\frac{r}{c}\right)^2 = 2\pi^2 S \tag{12–8}$$

12–4 Stable Lubrication

The difference between boundary and hydrodynamic lubrication can be explained by reference to Fig. 12–4. This plot of the change in the coefficient of friction versus the bearing characteristic $\mu N/P$ was obtained by the McKee brothers in an actual test of friction.[2] The plot is important because it defines stability of lubrication and helps us to understand hydrodynamic and boundary, or thin-film, lubrication.

Recall Petroff's bearing model in the form of Eq. (12–6) predicts that f is proportional to $\mu N/P$, that is, a straight line from the origin in the first quadrant. On the coordinates of Fig. 12–4 the locus to the right of point C is an example. Petroff's model presumes thick-film lubrication, that is, no metal-to-metal contact, the surfaces being completely separated by a lubricant film.

The McKee abscissa was ZN/P (centipoise × rev/min/psi) and the value of abscissa B in Fig. 12–4 was 30. The corresponding $\mu N/P$ (reyn × rev/s/psi) is $0.33(10^{-6})$. Designers keep $\mu N/P \geq 1.7(10^{-6})$, which corresponds to $ZN/P \geq 150$. A design constraint to keep thick-film lubrication is to be sure that

$$\frac{\mu N}{P} \geq 1.7(10^{-6}) \tag{a}$$

Suppose we are operating to the right of line BA and something happens, say, an increase in lubricant temperature. This results in a lower viscosity and hence a smaller value of $\mu N/P$. The coefficient of friction decreases, not as much heat is generated in shearing the lubricant, and consequently the lubricant temperature drops. Thus the region to the right of line BA defines *stable lubrication* because variations are self-correcting.

To the left of line BA, a decrease in viscosity would increase the friction. A temperature rise would ensue, and the viscosity would be reduced still more. The result would be compounded. Thus the region to the left of line BA represents *unstable lubrication*.

It is also helpful to see that a small viscosity, and hence a small $\mu N/P$, means that the lubricant film is very thin and that there will be a greater possibility of some metal-to-metal contact, and hence of more friction. Thus, point C represents what is probably the beginning of metal-to-metal contact as $\mu N/P$ becomes smaller.

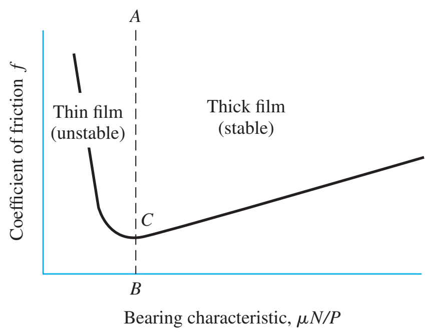


Figure 12–4

The variation of the coefficient of friction f with $\mu N/P$.

[2] S. A. McKee and T. R. McKee, "Journal Bearing Friction in the Region of Thin Film Lubrication," *SAE J.*, vol. 31, 1932, pp. (T)371–377.

12–5 Thick-Film Lubrication

Let us now examine the formation of a lubricant film in a journal bearing. Figure 12–5*a* shows a journal that is just beginning to rotate in a clockwise direction. Under starting conditions, the bearing will be dry, or at least partly dry, and hence the journal will climb or roll up the right side of the bearing as shown in Fig. 12–5*a*.

Now suppose a lubricant is introduced into the top of the bearing as shown in Fig. 12–5*b*. The action of the rotating journal is to pump the lubricant around the bearing in a clockwise direction. The lubricant is pumped into a wedge-shaped space and forces the journal over to the other side. A *minimum film thickness* h_0 occurs, not at the bottom of the journal, but displaced clockwise from the bottom as in Fig. 12–5*b*. This is explained by the fact that a film pressure in the converging half of the film reaches a maximum somewhere to the left of the bearing center.

Figure 12–5 shows how to decide whether the journal, under hydrodynamic lubrication, is eccentrically located on the right or on the left side of the bearing. Visualize the journal beginning to rotate. Find the side of the bearing upon which the journal tends to roll. Then, if the lubrication is hydrodynamic, mentally place the journal on the opposite side.

The nomenclature of a journal bearing is shown in Fig. 12–6. The dimension c is the *radial clearance* and is the difference in the radii of the bushing and journal. In Fig. 12–6 the center of the journal is at O and the center of the bearing at O'. The

Figure 12–5

Formation of a film.

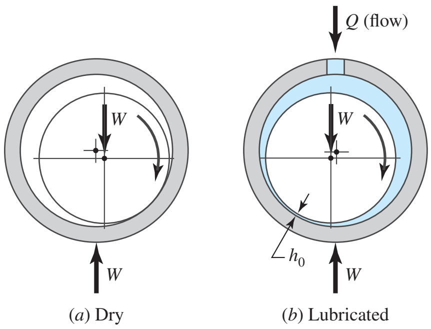
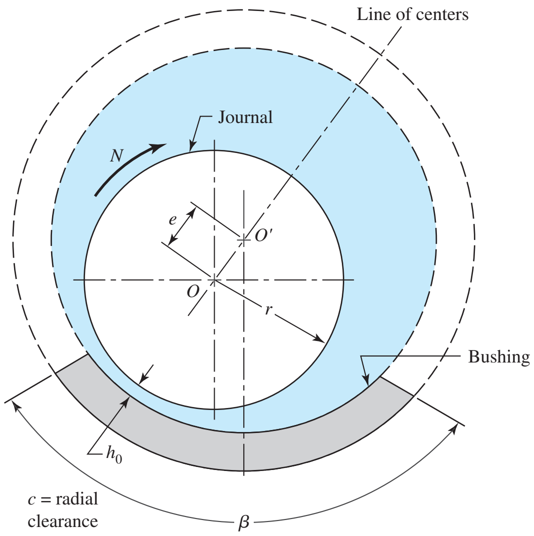


Figure 12–6

Nomenclature of a partial journal bearing.



distance between these centers is the *eccentricity* and is denoted by e. The *minimum film thickness* is designated by h_0, and it occurs at the line of centers. The film thickness at any other point is designated by h. We also define an *eccentricity ratio* ϵ as

$$\epsilon = \frac{e}{c}$$

The bearing shown in the figure is known as a *partial bearing*. If the radius of the bushing is the same as the radius of the journal, it is known as a *fitted bearing*. If the bushing encloses the journal, as indicated by the dashed lines, it becomes a *full bearing*. The angle β describes the angular length of a partial bearing. For example, a 120° partial bearing has the angle β equal to 120°.

12–6 Hydrodynamic Theory

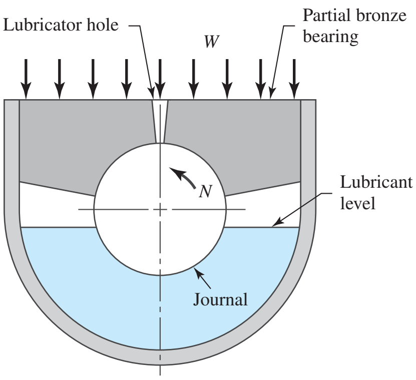
The present theory of hydrodynamic lubrication originated in the laboratory of Beauchamp Tower in the early 1880s in England. Tower had been employed to study the friction in railroad journal bearings and learn the best methods of lubricating them. It was an accident or error, during the course of this investigation, that prompted Tower to look at the problem in more detail and that resulted in a discovery that eventually led to the development of the theory.

Figure 12–7 is a schematic drawing of the journal bearing that Tower investigated. It is a partial bearing, having a diameter of 4 in, a length of 6 in, and a bearing arc of 157°, and having bath-type lubrication, as shown. The coefficients of friction obtained by Tower in his investigations on this bearing were quite low, which is now not surprising. After testing this bearing, Tower later drilled a $\frac{1}{2}$-in-diameter lubricator hole through the top. But when the apparatus was set in motion, oil flowed out of this hole. In an effort to prevent this, a cork stopper was used, but this popped out, and so it was necessary to drive a wooden plug into the hole. When the wooden plug was pushed out too, Tower, at this point, undoubtedly realized that he was on the verge of discovery. A pressure gauge connected to the hole indicated a pressure of more than twice the unit bearing load. Finally, he investigated the bearing film pressures in detail throughout the bearing width and length and reported a distribution similar to that of Fig. 12–8.[3]

The results obtained by Tower had such regularity that Osborne Reynolds concluded that there must be a definite equation relating the friction, the pressure, and

Figure 12–7

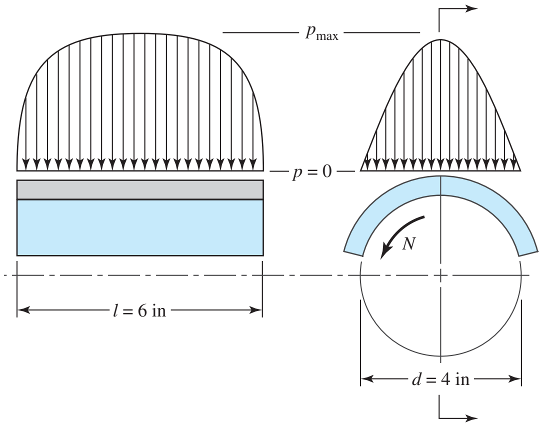
Schematic representation of the partial bearing used by Tower.



[3]Beauchamp Tower, "First Report on Friction Experiments," *Proc. Inst. Mech. Eng.,* November 1883, pp. 632–666; "Second Report," ibid., 1885, pp. 58–70; "Third Report," ibid., 1888, pp. 173–205; "Fourth Report," ibid., 1891, pp. 111–140.

Figure 12–8

Approximate pressure-distribution curves obtained by Tower.



the velocity. The present mathematical theory of lubrication is based upon Reynolds' work following the experiment by Tower.[4] The original differential equation, developed by Reynolds, was used by him to explain Tower's results. The solution is a challenging problem that has interested many investigators ever since then, and it is still the starting point for lubrication studies.

Reynolds pictured the lubricant as adhering to both surfaces and being pulled by the moving surface into a narrowing, wedge-shaped space so as to create a fluid or film pressure of sufficient intensity to support the bearing load. One of the important simplifying assumptions resulted from Reynolds' realization that the fluid films were so thin in comparison with the bearing radius that the curvature could be neglected. This enabled him to replace the curved partial bearing with a flat bearing, called a *plane slider bearing*. Other assumptions made were:

1 The lubricant obeys Newton's viscous effect, Eq. (12–1).

2 The forces due to the inertia of the lubricant are neglected.

3 The lubricant is assumed to be incompressible.

4 The viscosity is assumed to be constant throughout the film.

5 The pressure does not vary in the axial direction.

Figure 12–9*a* shows a journal rotating in the clockwise direction supported by a film of lubricant of variable thickness h on a partial bearing, which is fixed. We specify that the journal has a constant surface velocity U. Using Reynolds' assumption that curvature can be neglected, we fix a right-handed xyz reference system to the stationary bearing. We now make the following additional assumptions:

6 The bushing and journal extend infinitely in the z direction; this means there can be no lubricant flow in the z direction.

7 The film pressure is constant in the y direction. Thus the pressure depends only on the coordinate x.

8 The velocity of any particle of lubricant in the film depends only on the coordinates x and y.

We now select an element of lubricant in the film (Fig. 12–9*a*) of dimensions dx, dy, and dz, and compute the forces that act on the sides of this element. As shown in

[4]Osborne Reynolds, "Theory of Lubrication, Part I," *Phil. Trans. Roy. Soc. London,* 1886.

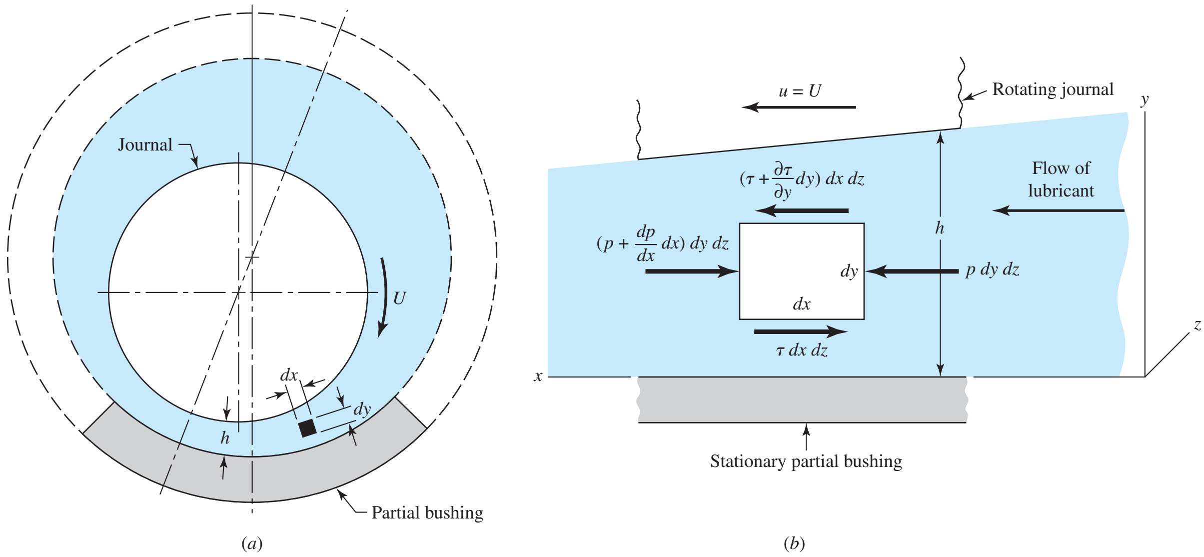


Figure 12–9

Fig. 12–9*b*, normal forces, due to the pressure, act upon the right and left sides of the element, and shear forces, due to the viscosity and to the velocity, act upon the top and bottom sides. Summing the forces in the x direction gives

$$\sum F_x = p\,dy\,dz - \left(p + \frac{dp}{dx}dx\right)dy\,dz - \tau\,dx\,dz + \left(\tau + \frac{\partial \tau}{\partial y}dy\right)dx\,dz = 0 \qquad (a)$$

This reduces to

$$\frac{dp}{dx} = \frac{\partial \tau}{\partial y} \qquad (b)$$

From Eq. (12–1), we have

$$\tau = \mu\frac{\partial u}{\partial y} \qquad (c)$$

where the partial derivative is used because the velocity u depends upon both x and y. Substituting Eq. (*c*) in Eq. (*b*), we obtain

$$\frac{dp}{dx} = \mu\frac{\partial^2 u}{\partial y^2} \qquad (d)$$

Holding x constant, we now integrate this expression twice with respect to y. This gives

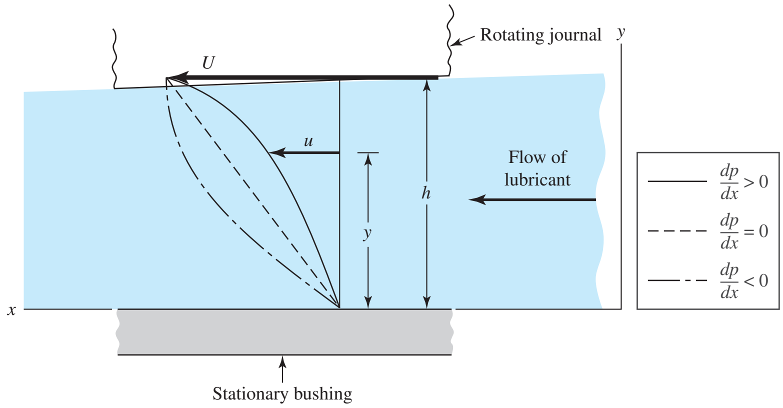
$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{dp}{dx}y + C_1$$
$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + C_1 y + C_2 \qquad (e)$$

Note that the act of holding x constant means that C_1 and C_2 can be functions of x. We now assume that there is no slip between the lubricant and the boundary surfaces. This gives two sets of boundary conditions for evaluating C_1 and C_2:

$$\text{At} \quad y = 0,\ u = 0$$
$$\text{At} \quad y = h,\ u = U \qquad (f)$$

Figure 12–10

Velocity of the lubricant.



Notice, in the second condition, that h is a function of x. Substituting these conditions in Eq. (e) and solving for C_1 and C_2 gives

$$C_1 = \frac{U}{h} - \frac{h}{2\mu}\frac{dp}{dx} \qquad C_2 = 0$$

or

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - hy) + \frac{U}{h}y \tag{12–9}$$

This equation gives the velocity distribution of the lubricant in the film as a function of the coordinate y and the pressure gradient dp/dx. The equation shows that the velocity distribution across the film (from $y = 0$ to $y = h$) is obtained by superposing a parabolic distribution onto a linear distribution. Figure 12–10 shows the superposition of these distributions to obtain the velocity for particular values of x and dp/dx. In general, the parabolic term may be additive or subtractive to the linear term, depending upon the sign of the pressure gradient. When the pressure is maximum, $dp/dx = 0$ and the velocity is

$$u = \frac{U}{h}y \tag{g}$$

which is a linear relation.

We next define Q as the volume of lubricant flowing in the x direction per unit time. By using a width of unity in the z direction, the volume may be obtained by the expression

$$Q = \int_0^h u\,dy \tag{h}$$

Substituting the value of u from Eq. (12–9) and integrating gives

$$Q = \frac{Uh}{2} - \frac{h^3}{12\mu}\frac{dp}{dx} \tag{i}$$

The next step uses the assumption of an incompressible lubricant and states that the flow is the same for any cross section. Thus

$$\frac{dQ}{dx} = 0$$

From Eq. (*i*),

$$\frac{dQ}{dx} = \frac{U}{2}\frac{dh}{dx} - \frac{d}{dx}\left(\frac{h^3}{12\mu}\frac{dp}{dx}\right) = 0$$

or

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6U\frac{dh}{dx} \tag{12–10}$$

which is the classical Reynolds equation for one-dimensional flow. It neglects side leakage, that is, flow in the z direction. A similar development is used when side leakage is not neglected. The resulting equation is

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 6U\frac{\partial h}{\partial x} \tag{12–11}$$

There is no general analytical solution to Eq. (12–11); approximate solutions have been obtained by using electrical analogies, mathematical summations, relaxation methods, and numerical and graphical methods. One of the important solutions is due to Sommerfeld[5] and may be expressed in the form

$$\frac{r}{c}f = \phi\left[\left(\frac{r}{c}\right)^2\frac{\mu N}{P}\right] \tag{12–12}$$

where ϕ indicates a functional relationship. Sommerfeld found the functions for half-bearings and full bearings by using the assumption of no side leakage.

12–7 Design Considerations

We may distinguish between two groups of variables in the design of sliding bearings. In the first group are those whose values either are given or are under the control of the designer. These are:

1. The viscosity μ
2. The load per unit of projected bearing area, P
3. The speed N
4. The bearing dimensions r, c, β, and l

Of these four variables, the designer usually has no control over the speed, because it is specified by the overall design of the machine. Sometimes the viscosity is specified in advance, as, for example, when the oil is stored in a sump and is used for lubricating and cooling a variety of bearings. The remaining variables, and sometimes the viscosity, may be controlled by the designer and are therefore the *decisions* the designer makes. In other words, when these four decisions are made, the design is complete.

In the second group are the dependent variables. The designer cannot control these except indirectly by changing one or more of the first group. These are:

1. The coefficient of friction f
2. The temperature rise ΔT
3. The volume flow rate of oil Q
4. The minimum film thickness h_0

[5] A. Sommerfeld, "Zur Hydrodynamischen Theorie der Schmiermittel-Reibung" ("On the Hydrodynamic Theory of Lubrication"), *Z. Math. Physik,* vol. 50, 1904, pp. 97–155.

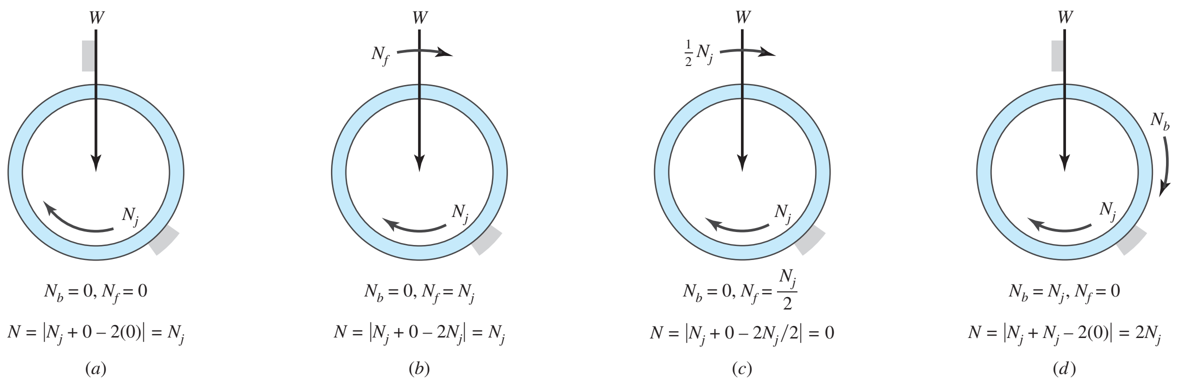


Figure 12–11

How the significant speed varies. (*a*) Common bearing case. (*b*) Load vector moves at the same speed as the journal. (*c*) Load vector moves at half journal speed, no load can be carried. (*d*) Journal and bushing move at same speed, load vector stationary, capacity halved.

This group of variables tells us how well the bearing is performing, and hence we may regard them as *performance factors*. Certain limitations on their values must be imposed by the designer to ensure satisfactory performance. These limitations are specified by the characteristics of the bearing materials and of the lubricant. The fundamental problem in bearing design, therefore, is to define satisfactory limits for the second group of variables and then to decide upon values for the first group such that these limitations are not exceeded.

Significant Angular Speed

In the next section we will examine several important charts relating key variables to the Sommerfeld number. To this point we have assumed that only the journal rotates and it is the journal rotational speed that is used in the Sommerfeld number. It has been discovered that the angular speed N that is significant to hydrodynamic film bearing performance is[6]

$$N = |N_j + N_b - 2N_f| \tag{12–13}$$

where N_j = journal angular speed, rev/s
N_b = bearing angular speed, rev/s
N_f = load vector angular speed, rev/s

When determining the Sommerfeld number for a general bearing, use Eq. (12–13) when entering N. Figure 12–11 shows several situations for determining N.

Trumpler's Design Criteria for Journal Bearings

Because the bearing assembly creates the lubricant pressure to carry a load, it reacts to loading by changing its eccentricity, which reduces the minimum film thickness h_0 until the load is carried. What is the limit of smallness of h_0? Close examination reveals that the moving adjacent surfaces of the journal and bushing are not smooth but consist of a series of asperities that pass one another, separated by a lubricant film. In starting

[6]Paul Robert Trumpler, *Design of Film Bearings,* Macmillan, New York, 1966, pp. 103–119.

a bearing under load from rest there is metal-to-metal contact and surface asperities are broken off, free to move and circulate with the oil. Unless a filter is provided, this debris accumulates. Such particles have to be free to tumble at the section containing the minimum film thickness without snagging in a togglelike configuration, creating additional damage and debris. Trumpler, an accomplished bearing designer, provides a throat of at least 200 μ in to pass particles from ground surfaces.[7] He also provides for the influence of size (tolerances tend to increase with size) by stipulating

$$h_0 \geq 0.0002 + 0.000\,04d \text{ in} \tag{a}$$

where d is the journal diameter in inches.

A lubricant is a mixture of hydrocarbons that reacts to increasing temperature by vaporizing the lighter components, leaving behind the heavier. This process (bearings have lots of time) slowly increases the viscosity of the remaining lubricant, which increases heat generation rate and elevates lubricant temperatures. This sets the stage for future failure. For light oils, Trumpler limits the maximum film temperature $T_{\max}$ to

$$T_{\max} \leq 250°\text{F} \tag{b}$$

Some oils can operate at slightly higher temperatures. Always check with the lubricant manufacturer.

A journal bearing often consists of a ground steel journal working against a softer, usually nonferrous, bushing. In starting under load there is metal-to-metal contact, abrasion, and the generation of wear particles, which, over time, can change the geometry of the bushing. The starting load divided by the projected area is limited to

$$\frac{W_{st}}{lD} \leq 300 \text{ psi} \tag{c}$$

If the load on a journal bearing is suddenly increased, the increase in film temperature in the annulus is immediate. Since ground vibration due to passing trucks, trains, and earth tremors is often present, Trumpler used a design factor of 2 or more on the running load, but not on the starting load of Eq. (c):

$$n_d \geq 2 \tag{d}$$

Many of Trumpler's designs are operating today, long after his consulting career was over; clearly they constitute good advice to the beginning designer.

12–8 The Relations of the Variables

Before proceeding to the problem of design, it is necessary to establish the relationships between the variables. Albert A. Raimondi and John Boyd, of Westinghouse Research Laboratories, used an iteration technique to solve Reynolds' equation on the digital computer.[8] This is the first time such extensive data have been available for use by designers, and consequently we shall employ them in this book.[9]

[7]Op. cit., pp. 192–194.

[8]A. A. Raimondi and John Boyd, "A Solution for the Finite Journal Bearing and Its Application to Analysis and Design, Parts I, II, and III," *Trans. ASLE,* vol. 1, no. 1, in *Lubrication Science and Technology,* Pergamon, New York, 1958, pp. 159–209.

[9]See also the earlier companion paper, John Boyd and Albert A. Raimondi, "Applying Bearing Theory to the Analysis and Design of Journal Bearings, Parts I and II," *J. Appl. Mechanics,* vol. 73, 1951, pp. 298–316.

The Raimondi and Boyd papers were published in three parts and contain 45 detailed charts and 6 tables of numerical information. In all three parts, charts are used to define the variables for length-diameter (l/d) ratios of 1:4, 1:2, and 1 and for beta angles of 60 to 360°. Under certain conditions the solution to the Reynolds equation gives negative pressures in the diverging portion of the oil film. Since a lubricant cannot usually support a tensile stress, Part III of the Raimondi-Boyd papers assumes that the oil film is ruptured when the film pressure becomes zero. Part III also contains data for the infinitely long bearing; since it has no ends, this means that there is no side leakage. The charts appearing in this book are from Part III of the papers, and are for full journal bearings ($\beta = 360°$) only. Space does not permit the inclusion of charts for partial bearings. This means that you must refer to the charts in the original papers when beta angles of less than 360° are desired. The notation is very nearly the same as in this book, and so no problems should arise.

Viscosity Charts (Figs. 12–12 to 12–14)

One of the most important assumptions made in the Raimondi-Boyd analysis is that *viscosity of the lubricant is constant as it passes through the bearing*. But since work is done on the lubricant during this flow, the temperature of the oil is higher when it leaves the loading zone than it was on entry. And the viscosity charts clearly indicate that the viscosity drops off significantly with a rise in temperature. Since the analysis is based on a constant viscosity, our problem now is to determine the value of viscosity to be used in the analysis.

Some of the lubricant that enters the bearing emerges as a side flow, which carries away some of the heat. The balance of the lubricant flows through the load-bearing zone and carries away the balance of the heat generated. In determining the viscosity to be used we shall employ a temperature that is the average of the inlet and outlet temperatures, or

$$T_{av} = T_1 + \frac{\Delta T}{2} \tag{12–14}$$

where T_1 is the inlet temperature and ΔT is the temperature rise of the lubricant from inlet to outlet. Of course, the viscosity used in the analysis must correspond to T_{av}.

Viscosity varies considerably with temperature in a nonlinear fashion. The ordinates in Figs. 12–12 to 12–14 are not logarithmic, as the decades are of differing vertical length. These graphs represent the temperature versus viscosity functions for common grades of lubricating oils in both customary engineering and SI units. We have the temperature versus viscosity function only in graphical form, unless curve fits are developed. See Table 12–1.

One of the objectives of lubrication analysis is to determine the oil outlet temperature when the oil and its inlet temperature are specified. This is a trial-and-error type of problem. In an analysis, the temperature rise will first be estimated. This allows for the viscosity to be determined from the chart. With the value of the viscosity, the analysis is performed where the temperature rise is then computed. With this, a new estimate of the temperature rise is established. This process is continued until the estimated and computed temperatures agree.

To illustrate, suppose we have decided to use SAE 30 oil in an application in which the oil inlet temperature is $T_1 = 180°F$. We begin by estimating that the

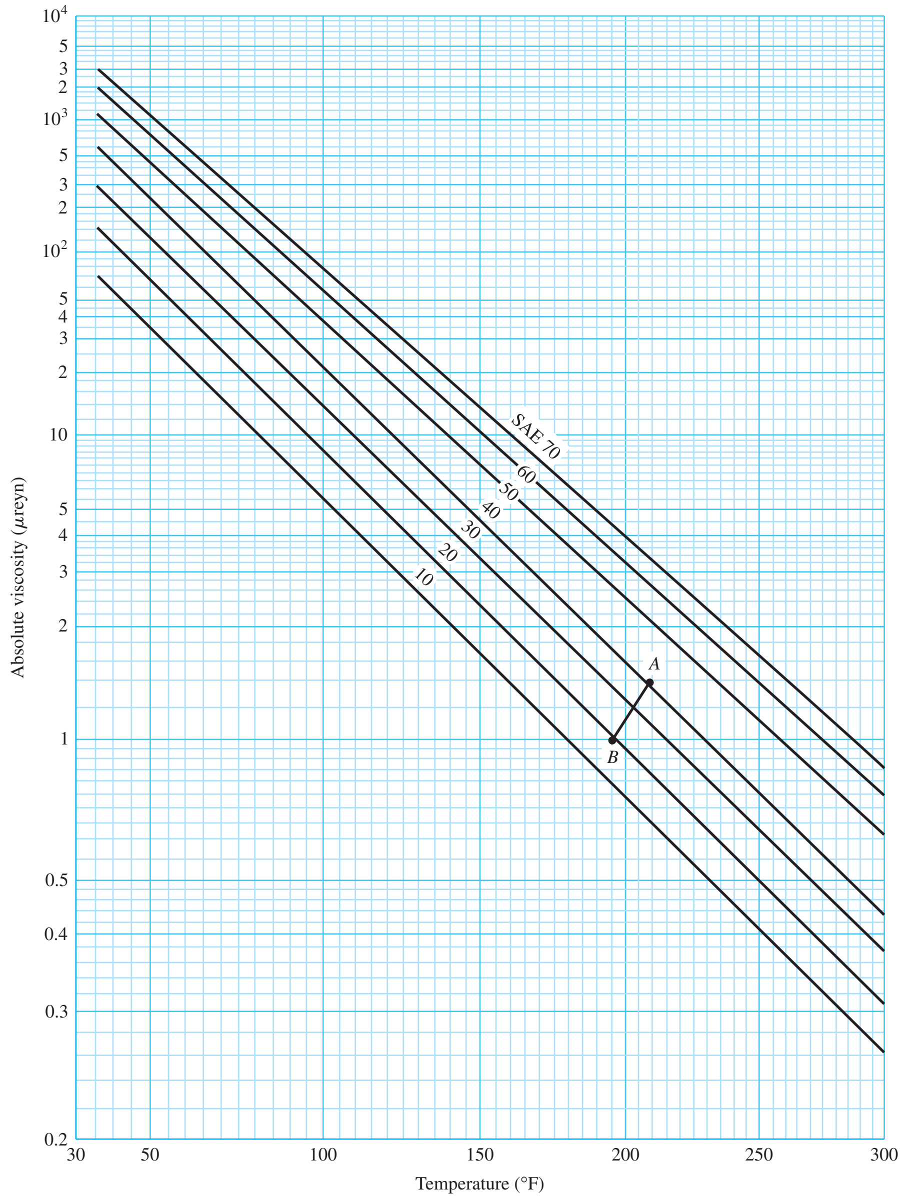


Figure 12–12

Viscosity–temperature chart in U.S. customary units. *(Raimondi and Boyd.)*

temperature rise will be $\Delta T = 30°\text{F}$. Then, from Eq. (12–14),

$$T_{av} = T_1 + \frac{\Delta T}{2} = 180 + \frac{30}{2} = 195°\text{F}$$

From Fig. 12–12 we follow the SAE 30 line and find that $\mu = 1.40\ \mu\text{reyn}$ at 195°F. So we use this viscosity (in an analysis to be explained in detail later) and find that the temperature rise is actually $\Delta T = 54°\text{F}$. Thus Eq. (12–14) gives

$$T_{av} = 180 + \frac{54}{2} = 207°\text{F}$$

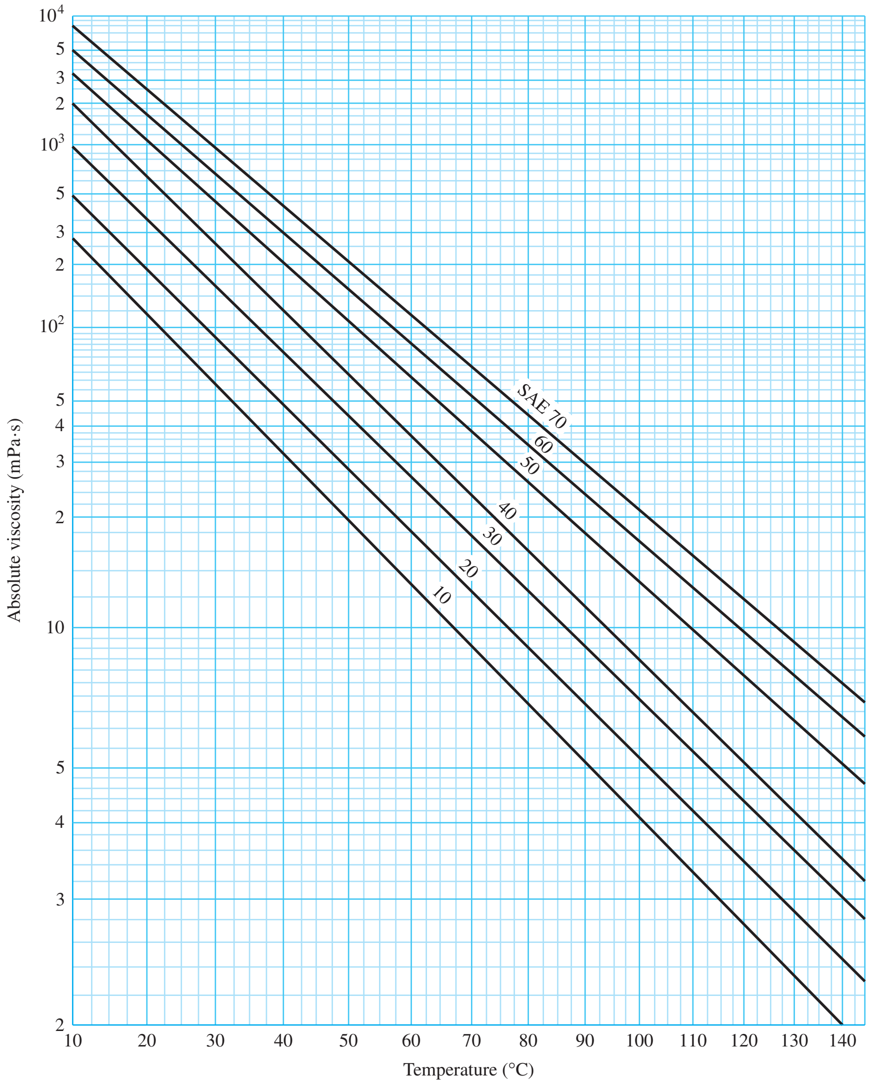


Figure 12–13

Viscosity–temperature chart in SI units. *(Adapted from Fig. 12–12.)*

This corresponds to point A on Fig. 12–12, which is above the SAE 30 line and indicates that the viscosity used in the analysis was too high.

For a second guess, try $\mu = 1.00$ μreyn. Again we run through an analysis and this time find that $\Delta T = 30°\text{F}$. This gives an average temperature of

$$T_{av} = 180 + \frac{30}{2} = 195°\text{F}$$

and locates point B on Fig. 12–12.

If points A and B are fairly close to each other and on opposite sides of the SAE 30 line, a straight line can be drawn between them with the intersection locating the correct values of viscosity and average temperature to be used in the analysis. For this illustration, we see from the viscosity chart that they are $T_{av} = 203°\text{F}$ and $\mu = 1.20$ μreyn.

Figure 12–14

Chart for multiviscosity lubricants. This chart was derived from known viscosities at two points, 100 and 210°F, and the results are believed to be correct for other temperatures.

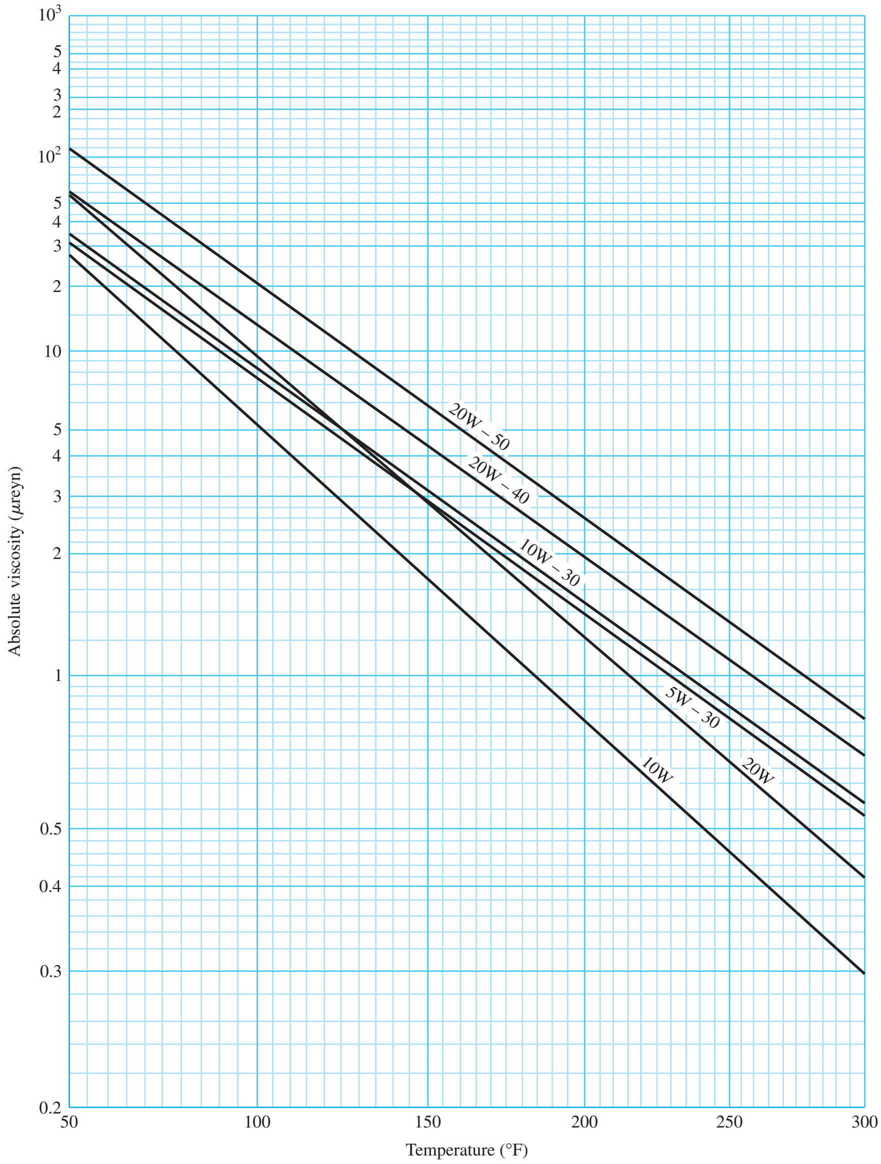


Table 12–1

Curve Fits* to Approximate the Viscosity versus Temperature Functions for SAE Grades 10 to 60

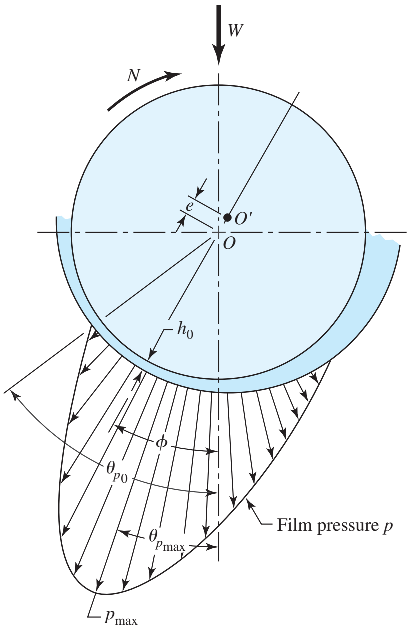
Source: A. S. Seireg and S. Dandage, "Empirical Design Procedure for the Thermodynamic Behavior of Journal Bearings," *J. Lubrication Technology,* vol. 104, April 1982, pp. 135–148.

Oil Grade, SAE	Viscosity μ_0, reyn	Constant b, °F
10	$0.0158(10^{-6})$	1157.5
20	$0.0136(10^{-6})$	1271.6
30	$0.0141(10^{-6})$	1360.0
40	$0.0121(10^{-6})$	1474.4
50	$0.0170(10^{-6})$	1509.6
60	$0.0187(10^{-6})$	1564.0

*$\mu = \mu_0 \exp\,[b/(T + 95)]$, T in °F.

Figure 12–15

Polar diagram of the film–pressure distribution showing the notation used. *(Raimondi and Boyd.)*



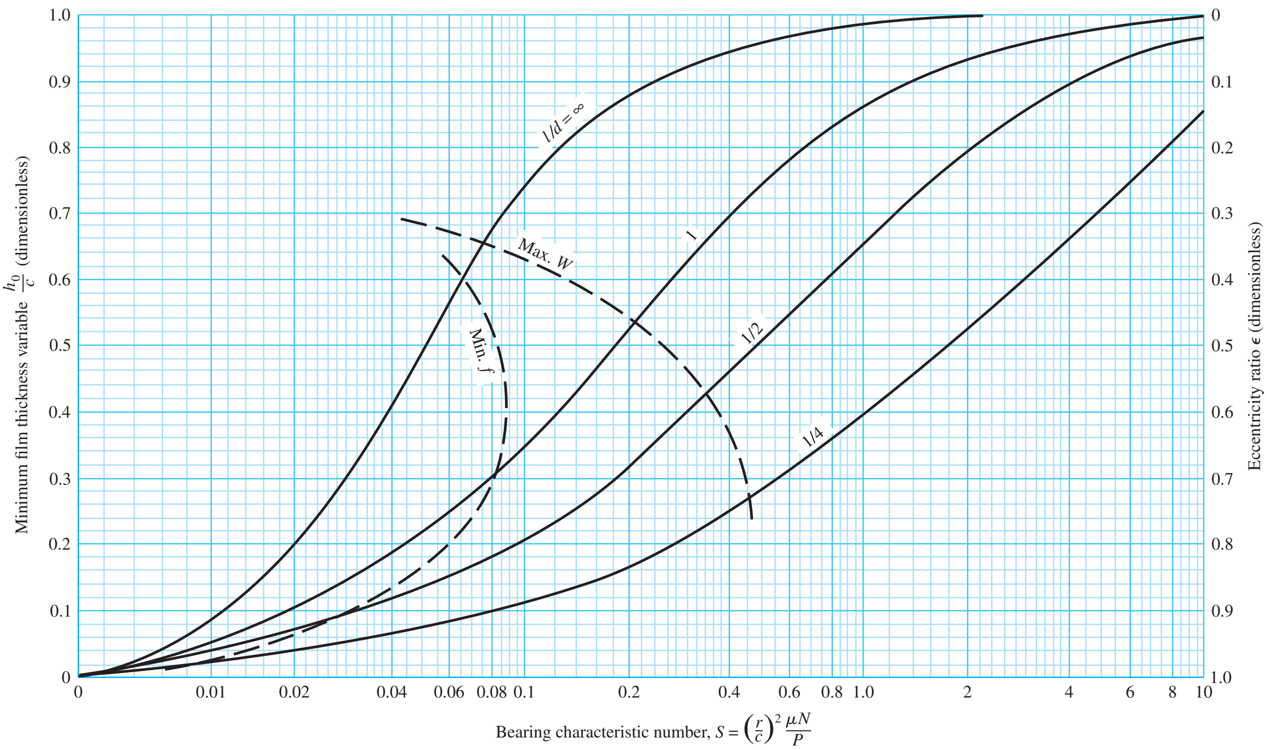


Figure 12–16

Chart for minimum film thickness variable and eccentricity ratio. The left boundary of the zone defines the optimal h_0 for minimum friction; the right boundary is optimum h_0 for load. *(Raimondi and Boyd.)*

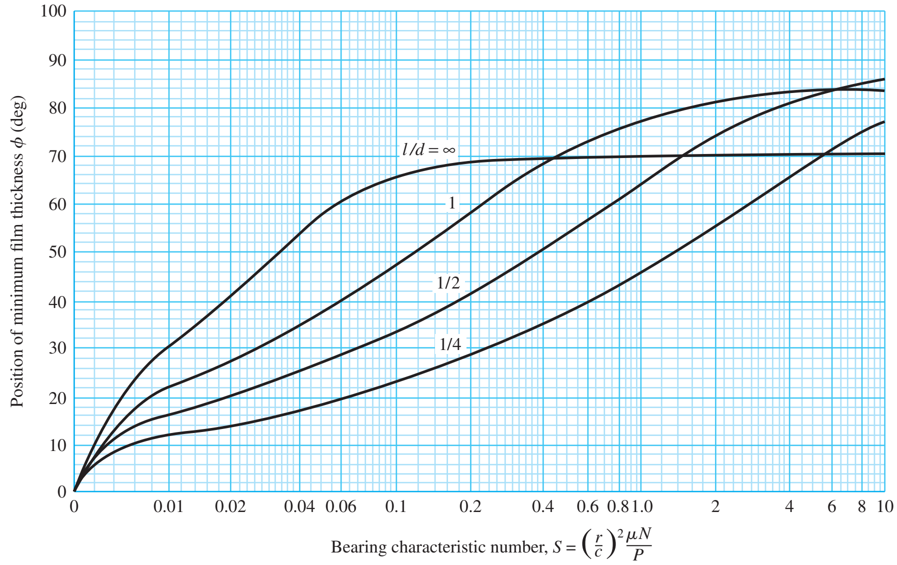


Figure 12–17

Chart for determining the position of the minimum film thickness h_0. *(Raimondi and Boyd.)*

The remaining charts from Raimondi and Boyd relate several variables to the Sommerfeld number. These variables are

Minimum film thickness (Figs. 12–16 and 12–17)
Coefficient of friction (Fig. 12–18)
Lubricant flow (Figs. 12–19 and 12–20)
Film pressure (Figs. 12–21 and 12–22)

Figure 12–15 shows the notation used for the variables. We will describe the use of these curves in a series of four examples using the same set of given parameters.

Minimum Film Thickness

In Fig. 12–16, *the minimum film thickness* variable h_0/c and *eccentricity ratio* $\epsilon = e/c$ are plotted against the Sommerfeld number S with contours for various values of l/d. The corresponding angular position of the minimum film thickness is found in Fig. 12–17.

EXAMPLE 12–1

Determine h_0 and e using the following given parameters: $\mu = 4\ \mu$reyn, $N = 30$ rev/s, $W = 500$ lbf (bearing load), $r = 0.75$ in, $c = 0.0015$ in, and $l = 1.5$ in.

Solution

The nominal bearing pressure (in projected area of the journal) is

$$P = \frac{W}{2rl} = \frac{500}{2(0.75)1.5} = 222 \text{ psi}$$

The Sommerfeld number is, from Eq. (12–7), where $N = N_j = 30$ rev/s,

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu N}{P}\right) = \left(\frac{0.75}{0.0015}\right)^2 \left[\frac{4(10^{-6})30}{222}\right] = 0.135$$

Also, $l/d = 1.50/[2(0.75)] = 1$. Entering Fig. 12–16 with $S = 0.135$ and $l/d = 1$ gives $h_0/c = 0.42$ and $\epsilon = 0.58$. The quantity h_0/c is called the *minimum film thickness* *variable*. Since $c = 0.0015$ in, the minimum film thickness h_0 is

$$h_0 = 0.42(0.0015) = 0.000\,63 \text{ in}$$

We can find the angular location ϕ of the minimum film thickness from the chart of Fig. 12–17. Entering with $S = 0.135$ and $l/d = 1$ gives $\phi = 53°$.

The eccentricity ratio is $\epsilon = e/c = 0.58$. This means the eccentricity e is

$$e = 0.58(0.0015) = 0.000\,87 \text{ in}$$

Note that if the journal is centered in the bushing, $e = 0$ and $h_0 = c$, corresponding to a very light (zero) load. Since $e = 0$, $\epsilon = 0$. As the load is increased the journal displaces downward; the limiting position is reached when $h_0 = 0$ and $e = c$, that is, when the journal touches the bushing. For this condition the eccentricity ratio is unity. Since $h_0 = c - e$, dividing both sides by c, we have

$$\frac{h_0}{c} = 1 - \epsilon$$

Design optima are sometimes *maximum load*, which is a load-carrying characteristic of the bearing, and sometimes *minimum parasitic power loss* or *minimum coefficient of friction*. Dashed lines appear on Fig. 12–16 for maximum load and minimum coefficient of friction, so you can easily favor one of maximum load or minimum coefficient of friction, but not both. The zone between the two dashed-line contours might be considered a desirable location for a design point.

Coefficient of Friction

The friction chart, Fig. 12–18, has the *friction variable* $(r/c)f$ plotted against Sommerfeld number S with contours for various values of the l/d ratio.

EXAMPLE 12–2

Using the parameters given in Ex. 12–1, determine the coefficient of friction, the torque to overcome friction, and the power loss to friction.

Solution

We enter Fig. 12–18 with $S = 0.135$ and $l/d = 1$ and find $(r/c)f = 3.50$. The coefficient of friction f is

$$f = 3.50\, c/r = 3.50(0.0015/0.75) = 0.0070$$

The friction torque on the journal is

$$T = f\,Wr = 0.007(500)0.75 = 2.62 \text{ lbf} \cdot \text{in}$$

The power loss in horsepower is

$$(hp)_{\text{loss}} = \frac{TN}{1050} = \frac{2.62(30)}{1050} = 0.075 \text{ hp}$$

or, expressed in Btu/s,

$$H = \frac{2\pi TN}{778(12)} = \frac{2\pi(2.62)(30)}{778(12)} = 0.0529 \text{ Btu/s}$$

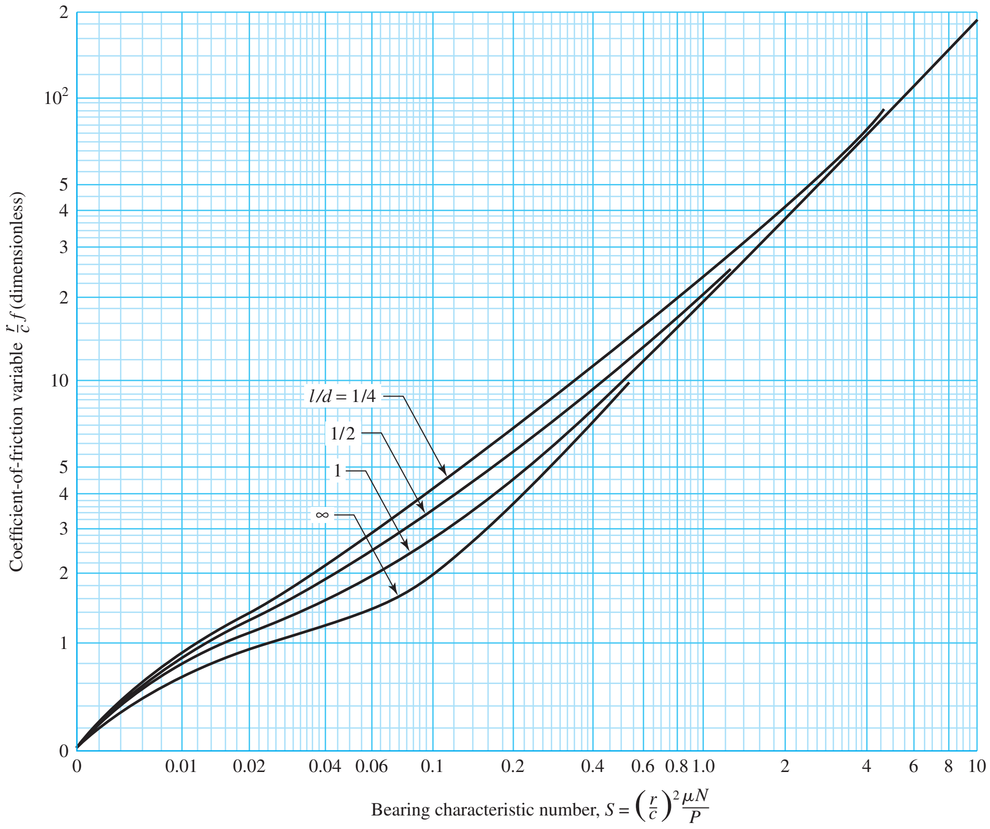


Figure 12–18

Chart for coefficient-of-friction variable; note that Petroff's equation is the asymptote. *(Raimondi and Boyd.)*

Lubricant Flow

Figures 12–19 and 12–20 are used to determine the lubricant flow and side flow.

EXAMPLE 12–3

Continuing with the parameters of Ex. 12–1, determine the total volumetric flow rate Q and the side flow rate Q_s.

Solution

To estimate the lubricant flow, enter Fig. 12–19 with $S = 0.135$ and $l/d = 1$ to obtain $Q/(rcNl) = 4.28$. The total volumetric flow rate is

$$Q = 4.58rcNl = 4.28(0.75)0.0015(30)1.5 = 0.217 \text{ in}^3/\text{s}$$

From Fig. 12–20 we find the *flow ratio* $Q_s/Q = 0.655$ and Q_s is

$$Q_s = 0.655Q = 0.655(0.217) = 0.142 \text{ in}^3/\text{s}$$

Figure 12–19

Chart for flow variable.
Note: Not for pressure-fed bearings. *(Raimondi and Boyd.)*

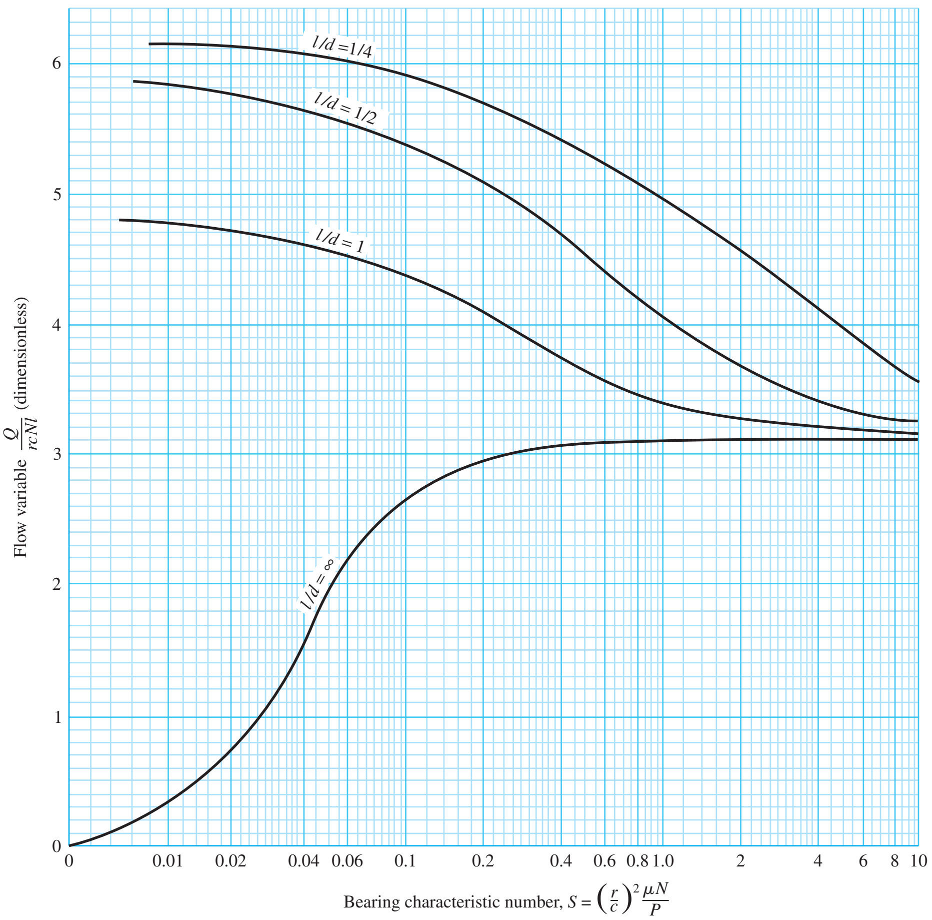
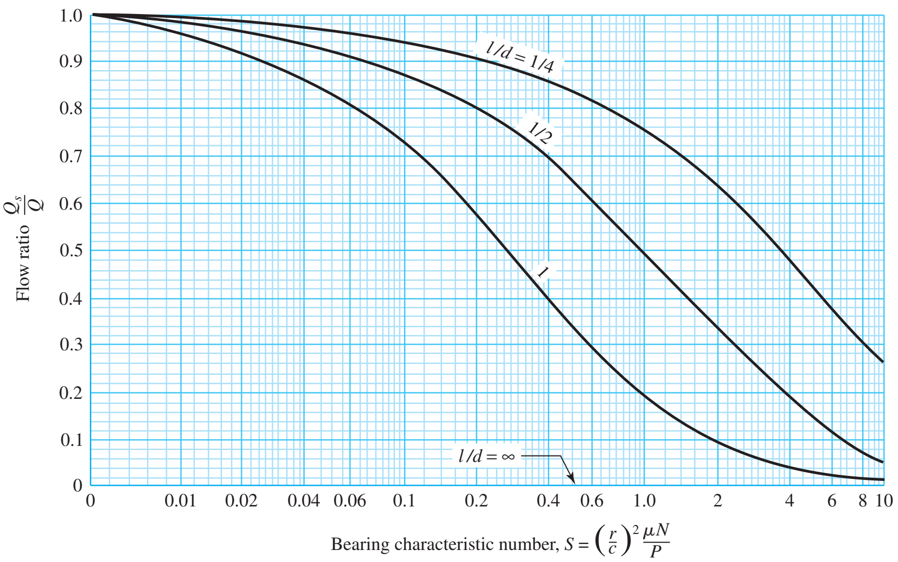


Figure 12–20

Chart for determining the ratio of side flow to total flow. *(Raimondi and Boyd.)*



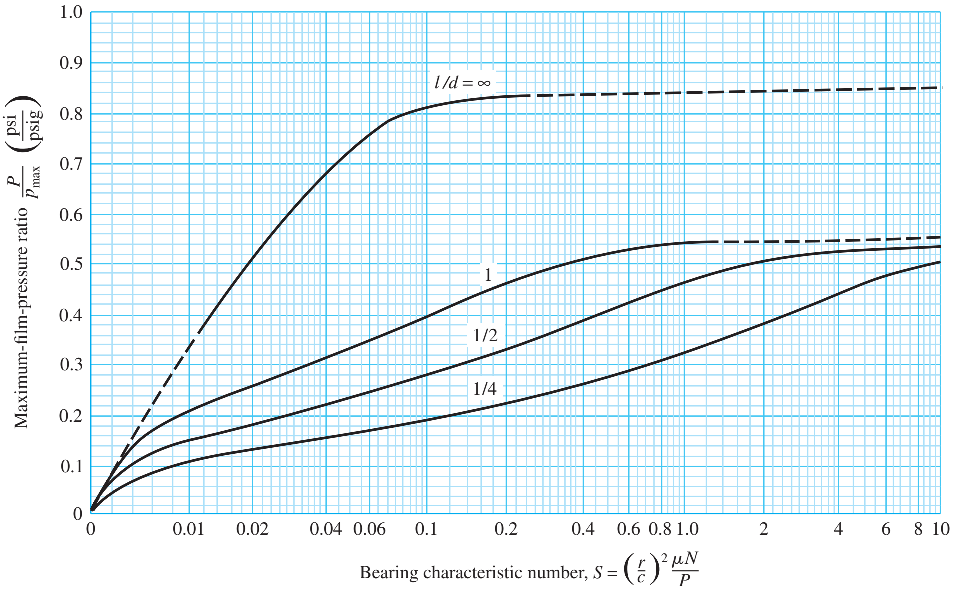


Figure 12–21

Chart for determining the maximum film pressure. *Note:* Not for pressure-fed bearings. *(Raimondi and Boyd.)*

The side leakage Q_s is from the lower part of the bearing, where the internal pressure is above atmospheric pressure. The leakage forms a fillet at the journal-bushing external junction, and it is carried by journal motion to the top of the bushing, where the internal pressure is below atmospheric pressure and the gap is much larger, to be "sucked in" and returned to the lubricant sump. That portion of side leakage that leaks away from the bearing has to be made up by adding oil to the bearing sump periodically by maintenance personnel.

Film Pressure

The maximum pressure developed in the film can be estimated by finding the pressure ratio P/p_{max} from the chart in Fig. 12–21. The locations where the terminating and maximum pressures occur, as defined in Fig 12–15, are determined from Fig. 12–22.

EXAMPLE 12–4 Using the parameters given in Ex. 12–1, determine the maximum film pressure and the locations of the maximum and terminating pressures.

Solution Entering Fig. 12–21 with $S = 0.135$ and $l/d = 1$, we find $P/p_{max} = 0.42$. The maximum pressure p_{max} is therefore

$$p_{max} = \frac{P}{0.42} = \frac{222}{0.42} = 529 \text{ psi}$$

With $S = 0.135$ and $l/d = 1$, from Fig. 12–22, $\theta_{p_{max}} = 18.5°$ and the terminating position θ_{p_0} is 75°.

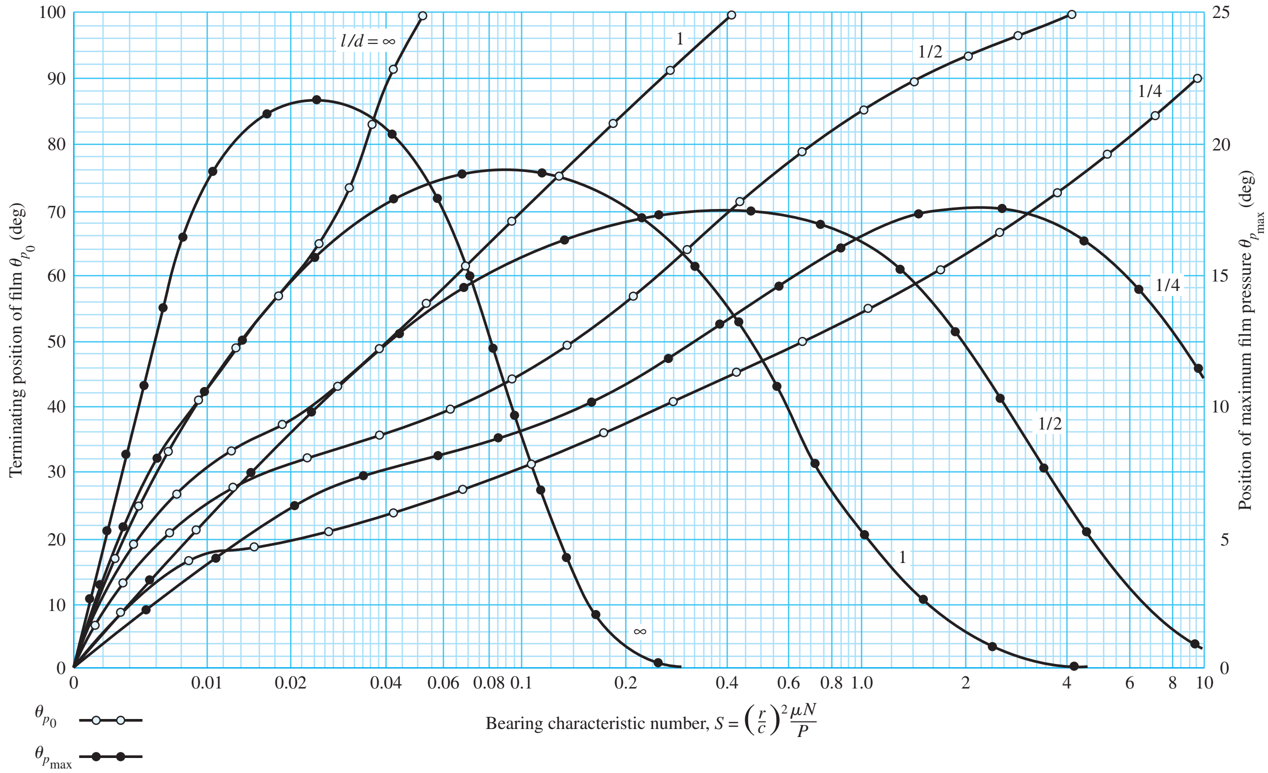


Figure 12–22

Chart for finding the terminating position of the lubricant film and the position of maximum film pressure. *(Raimondi and Boyd.)*

Examples 12–1 to 12–4 demonstrate how the Raimondi and Boyd charts are used. It should be clear that we do not have journal-bearing parametric relations as equations, but in the form of charts. Moreover, the examples were simple because the steady-state equivalent viscosity was given. We will now show how the average film temperature (and the corresponding viscosity) is found from energy considerations.

Lubricant Temperature Rise

The temperature of the lubricant rises until the rate at which work is done by the journal on the film through fluid shear is the same as the rate at which heat is transferred to the greater surroundings. The specific arrangement of the bearing plumbing affects the quantitative relationships. See Fig. 12–23. A lubricant sump (internal or external to the bearing housing) supplies lubricant at sump temperature T_s to the bearing annulus at temperature $T_s = T_1$. The lubricant passes once around the bushing and is delivered at a higher lubricant temperature $T_1 + \Delta T$ to the sump. Some of the lubricant leaks out of the bearing at a mixing-cup temperature of $T_1 + \Delta T/2$ and is returned to the sump. The sump may be a keyway-like groove in the bearing cap or a larger chamber up to half the bearing circumference. It can occupy "all" of the bearing cap of a split bearing. In such a bearing the side leakage occurs from the lower portion and is sucked back in, into the ruptured film arc. The sump could be well removed from the journal-bushing interface.

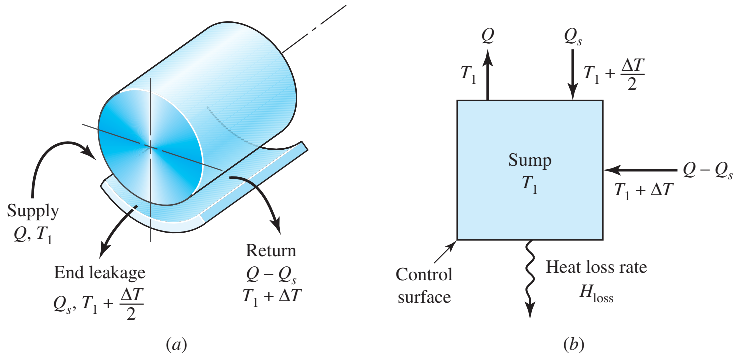


Figure 12–23

Schematic of a journal bearing with an external sump with cooling; lubricant makes one pass before returning to the sump.

Let

Q = volumetric oil-flow rate into the bearing, in^3/s
Q_s = volumetric side-flow leakage rate out of the bearing and to the sump, in^3/s
$Q - Q_s$ = volumetric oil-flow discharge from annulus to sump, in^3/s
T_1 = oil inlet temperature (equal to sump temperature T_s), °F
ΔT = temperature rise in oil between inlet and outlet, °F
ρ = lubricant density, lbm/in^3
C_p = specific heat capacity of lubricant, Btu/(lbm · °F)
J = Joulean heat equivalent, in · lbf/Btu
H = heat rate, Btu/s

Using the sump as a control region, we can write an enthalpy balance. Using T_1 as the datum temperature gives

$$H_{\text{loss}} = \rho C_p Q_s \Delta T/2 + \rho C_p (Q - Q_s)\Delta T = \rho C_p Q \Delta T\left(1 - 0.5\frac{Q_s}{Q}\right) \qquad (a)$$

The thermal energy loss at steady state H_{loss} is equal to the rate the journal does work on the film is $H_{\text{loss}} = \dot{W} = 2\pi TN/J$. The torque $T = fWr$, the load in terms of pressure is $W = 2Prl$, and multiplying numerator and denominator by the clearance c gives

$$H_{\text{loss}} = \frac{4\pi\, PrlNc}{J}\frac{rf}{c} \qquad (b)$$

Equating Eqs. (*a*) and (*b*) and rearranging results in

$$\frac{J\rho C_p\,\Delta T}{4\pi\, P} = \frac{rf/c}{(1 - 0.5Q_s/Q)[Q/(rcNl)]} \qquad (c)$$

For common petroleum lubricants $\rho = 0.0311$ lbm/in^3, $C_p = 0.42$ Btu/(lbm · °F), and $J = 778(12) = 9336$ in · lbf/Btu; therefore the left term of Eq. (*c*) is

$$\frac{J\rho C_p\,\Delta T}{4\pi\, P} = \frac{9336(0.0311)0.42\Delta T_F}{4\pi\, P_{\text{psi}}} = 9.70\frac{\Delta T_F}{P_{\text{psi}}}$$

thus

$$\frac{9.70\Delta T_F}{P_{\text{psi}}} = \frac{rf/c}{(1 - 0.5Q_s/Q)[Q/(rcN_j l)]} \qquad (12\text{–}15)$$

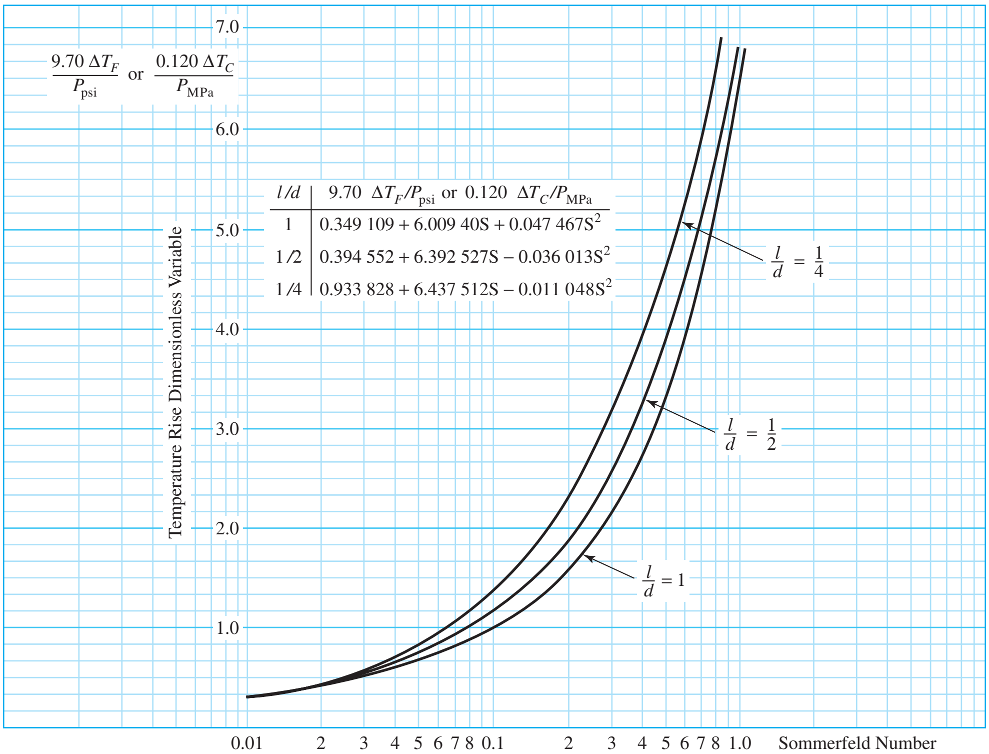


Figure 12–24

Figures 12–18, 12–19, and 12–20 combined to reduce iterative table look-up. *(Source: Chart based on work of Raimondi and Boyd boundary condition (2), i.e., no negative lubricant pressure developed. Chart is for full journal bearing using single lubricant pass, side flow emerges with temperature rise $\Delta T/2$, thru flow emerges with temperature rise ΔT, and entire flow is supplied at datum sump temperature.)*

where ΔT_F is the temperature rise in °F and P_{psi} is the bearing pressure in psi. The right side of Eq. (12–15) can be evaluated from Figs. 12–18, 12–19, and 12–20 for various Sommerfeld numbers and l/d ratios to give Fig. 12–24. It is easy to show that the left side of Eq. (12–15) can be expressed as $0.120\Delta T_C/P_{MPa}$ where ΔT_C is expressed in °C and the pressure P_{MPa} is expressed in MPa. The ordinate in Fig. 12–24 is either $9.70\,\Delta T_F/P_{psi}$ or $0.120\,\Delta T_C/P_{MPa}$, which is not surprising since both are dimensionless in proper units and *identical in magnitude*. Since solutions to bearing problems involve iteration and reading many graphs can introduce errors, Fig. 12–24 reduces three graphs to one, a step in the proper direction.

Interpolation

For l/d ratios other than the ones given in the charts, Raimondi and Boyd have provided the following interpolation equation

$$y = \frac{1}{(l/d)^3}\left[-\frac{1}{8}\left(1-\frac{l}{d}\right)\left(1-2\frac{l}{d}\right)\left(1-4\frac{l}{d}\right)y_\infty + \frac{1}{3}\left(1-2\frac{l}{d}\right)\left(1-4\frac{l}{d}\right)y_1 \right.$$
$$\left. -\frac{1}{4}\left(1-\frac{l}{d}\right)\left(1-4\frac{l}{d}\right)y_{1/2} + \frac{1}{24}\left(1-\frac{l}{d}\right)\left(1-2\frac{l}{d}\right)y_{1/4}\right] \qquad (12\text{–}16)$$

where y is the desired variable within the interval $\infty > l/d > \frac{1}{4}$ and y_∞, y_1, $y_{1/2}$, and $y_{1/4}$ are the variables corresponding to l/d ratios of ∞, 1, $\frac{1}{2}$, and $\frac{1}{4}$, respectively.

12–9 Steady-State Conditions in Self-Contained Bearings

The case in which the lubricant carries away all of the enthalpy increase from the journal-bushing pair has already been discussed. Bearings in which the warm lubricant stays within the bearing housing will now be addressed. These bearings are called *self-contained* bearings because the lubricant sump is within the bearing housing and the lubricant is cooled within the housing. These bearings are described as *pillow-block* or *pedestal* bearings. They find use on fans, blowers, pumps, and motors, for example. Integral to design considerations for these bearings is dissipating heat from the bearing housing to the surroundings at the same rate that enthalpy is being generated within the fluid film.

In a self-contained bearing the sump can be positioned as a keywaylike cavity in the bushing, the ends of the cavity not penetrating the end planes of the bushing. Film oil exits the annulus at about one-half of the relative peripheral speeds of the journal and bushing and slowly tumbles the sump lubricant, mixing with the sump contents. Since the film in the top "half" of the cap has cavitated, it contributes essentially nothing to the support of the load, but it does contribute friction. Bearing caps are in use in which the "keyway" sump is expanded peripherally to encompass the top half of the bearing. This reduces friction for the same load, but the included angle β of the bearing has been reduced to 180°. Charts for this case were included in the Raimondi and Boyd paper.

The heat given up by the bearing housing may be estimated from the equation

$$H_{\text{loss}} = \hbar_{\text{CR}} A (T_b - T_\infty) \tag{12–17}$$

where H_{loss} = heat dissipated, Btu/h

$\hbar_{\text{CR}}$ = combined overall coefficient of radiation and convection heat transfer, Btu/(h · ft^2 · °F)

A = surface area of bearing housing, ft^2

T_b = surface temperature of the housing, °F

T_∞ = ambient temperature, °F

The overall coefficient $\hbar_{\text{CR}}$ depends on the material, surface coating, geometry, even the roughness, the temperature difference between the housing and surrounding objects, and air velocity. After Karelitz,[10] and others, in ordinary industrial environments, the overall coefficient $\hbar_{\text{CR}}$ can be treated as a constant. Some representative values are

$$\hbar_{\text{CR}} = \begin{cases} 2\ \text{Btu/(h} \cdot \text{ft}^2 \cdot {}^\circ\text{F)} & \text{for still air} \\ 2.7\ \text{Btu/(h} \cdot \text{ft}^2 \cdot {}^\circ\text{F)} & \text{for shaft-stirred air} \\ 5.9\ \text{Btu/(h} \cdot \text{ft}^2 \cdot {}^\circ\text{F)} & \text{for air moving at 500 ft/min} \end{cases} \tag{12–18}$$

An expression similar to Eq. (12–17) can be written for the temperature difference $T_f - T_b$ between the lubricant film and the housing surface. This is possible because the bushing and housing are metal and very nearly isothermal. If one defines $\bar{T}_f$ as the *average* film temperature (halfway between the lubricant inlet temperature T_s and

[10] G. B. Karelitz, "Heat Dissipation in Self-Contained Bearings," *Trans. ASME,* Vol. 64, 1942, p. 463; D. C. Lemmon and E. R. Booser, "Bearing Oil-Ring Performance," *Trans. ASME,* J. Bas. Engin., Vol. 88, 1960, p. 327.

Table 12–2

Lubrication System	Conditions	Range of α
Oil ring	Moving air	1–2
	Still air	$\frac{1}{2}$–1
Oil bath	Moving air	$\frac{1}{2}$–1
	Still air	$\frac{1}{5}$–$\frac{2}{5}$

the outlet temperature $T_s + \Delta T$), then the following proportionality has been observed between $\bar{T}_f - T_b$ and the difference between the housing surface temperature and the ambient temperature, $T_b - T_\infty$:

$$\bar{T}_f - T_b = \alpha(T_b - T_\infty) \qquad (a)$$

where $\bar{T}_f$ is the average film temperature and α is a constant depending on the lubrication scheme and the bearing housing geometry. Equation (*a*) may be used to estimate the bearing housing temperature. Table 12–2 provides some guidance concerning suitable values of α. The work of Karelitz allows the broadening of the application of the charts of Raimondi and Boyd, to be applied to a variety of bearings beyond the natural circulation pillow-block bearing.

Solving Eq. (*a*) for T_b and substituting into Eq. (12–17) gives the bearing heat loss rate to the surroundings as

$$H_{\text{loss}} = \frac{\hbar_{\text{CR}} A}{1 + \alpha}(\bar{T}_f - T_\infty) \qquad (12\text{–}19a)$$

and rewriting Eq. (*a*) gives

$$T_b = \frac{\bar{T}_f + \alpha T_\infty}{1 + \alpha} \qquad (12\text{–}19b)$$

In beginning a steady-state analysis the average film temperature is unknown, hence the viscosity of the lubricant in a self-contained bearing is unknown. Finding the equilibrium temperatures is an iterative process wherein a trial average film temperature (and the corresponding viscosity) is used to compare the heat generation rate and the heat loss rate. An adjustment is made to bring these two heat rates into agreement. This can be done on paper with a tabular array to help adjust $\bar{T}_f$ to achieve equality between heat generation and loss rates. A root-finding algorithm can be used. Even a simple one can be programmed for a digital computer.

Because of the shearing action there is a uniformly distributed energy release in the lubricant that heats the lubricant as it works its way around the bearing. The temperature is uniform in the radial direction but increases from the sump temperature T_s by an amount ΔT during the lubricant pass. The exiting lubricant mixes with the sump contents, being cooled to sump temperature. The lubricant in the sump is cooled because the bushing and housing metal are at a nearly uniform lower temperature because of heat losses by convection and radiation to the surroundings at ambient temperature T_∞. In the usual configurations of such bearings, the bushing and housing metal temperature is approximately midway between the average film temperature $\bar{T}_f = T_s + \Delta T/2$ and the ambient temperature T_∞. The heat generation rate H_{gen}, at steady state, is equal to the work rate from the frictional torque T. Expressing this in Btu/h requires the conversion constants 2545 Btu/(hp · h) and 1050 (lbf · in)(rev/s)/hp results in $H_{gen} = 2545\, TN/1050$. Then from Eq. (*b*), Sec. 12–3, the torque is $T = 4\pi^2 r^3 l\mu/c$, resulting in

$$H_{gen} = \frac{2545}{1050}\frac{4\pi^2 r^3 l\mu N}{c}N = \frac{95.69\mu N^2 l r^3}{c} \tag{b}$$

Equating this to Eq. (12–19*a*) and solving for $\bar{T}_f$ gives

$$\bar{T}_f = T_\infty + 95.69(1+\alpha)\frac{\mu N^2 l r^3}{\hbar_{CR} A c} \tag{12–20}$$

EXAMPLE 12–5

Consider a pillow-block bearing with a keyway sump, whose journal rotates at 900 rev/min in shaft-stirred air at 70°F with $\alpha = 1$. The lateral area of the bearing is 40 in^2. The lubricant is SAE grade 20 oil. The gravity radial load is 100 lbf and the l/d ratio is unity. The bearing has a journal diameter of 2.000 + 0.000/−0.002 in, a bushing bore of 2.002 + 0.004/−0.000 in. For a minimum clearance assembly estimate the steady-state temperatures as well as the minimum film thickness and coefficient of friction.

Solution

The minimum radial clearance, c_{min}, is

$$c_{min} = \frac{2.002 - 2.000}{2} = 0.001 \text{ in}$$

$$P = \frac{W}{ld} = \frac{100}{(2)2} = 25 \text{ psi}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{1}{0.001}\right)^2 \frac{\mu'(15)}{10^6(25)} = 0.6\,\mu'$$

where μ' is viscosity in μreyn. The friction horsepower loss, $(\text{hp})_f$, is found as follows:

$$(\text{hp})_f = \frac{fWrN}{1050} = \frac{WNc}{1050}\frac{fr}{c} = \frac{100(900/60)0.001}{1050}\frac{fr}{c} = 0.001\,429\frac{fr}{c}\text{ hp}$$

The heat generation rate H_{gen}, in Btu/h, is

$$H_{gen} = 2545(\text{hp})_f = 2545(0.001\,429)fr/c = 3.637\,fr/c \text{ Btu/h}$$

From Eq. (12–19*a*) with $\hbar_{CR} = 2.7$ Btu/(h · ft^2 · °F), the rate of heat loss to the environment H_{loss} is

$$H_{loss} = \frac{\hbar_{CR} A}{\alpha + 1}(\bar{T}_f - 70) = \frac{2.7(40/144)}{(1+1)}(\bar{T}_f - 70) = 0.375(\bar{T}_f - 70)\text{ Btu/h}$$

Build a table as follows for trial values of $\bar{T}_f$ of 190 and 195°F:

Trial $\bar{T}_f$	μ'	S	fr/c	H_{gen}	H_{loss}
190	1.15	0.69	13.6	49.5	45.0
195	1.03	0.62	12.2	44.4	46.9

The temperature at which $H_{\text{gen}} = H_{\text{loss}} = 46.3$ Btu/h is 193.4°F. Rounding $\bar{T}_f$ to 193°F we find $\mu' = 1.08$ μreyn and $S = 0.6(1.08) = 0.65$. From Fig. 12–24, $9.70\Delta T_F/P = 4.25$°F/psi and thus

$$\Delta T_F = 4.25P/9.70 = 4.25(25)/9.70 = 11.0°\text{F}$$

$$T_1 = T_s = \bar{T}_f - \Delta T/2 = 193 - 11/2 = 187.5°\text{F}$$

$$T_{\max} = T_1 + \Delta T_F = 187.5 + 11 = 198.5°\text{F}$$

From Eq. (12–19*b*)

$$T_b = \frac{T_f + \alpha T_\infty}{1 + \alpha} = \frac{193 + (1)70}{1 + 1} = 131.5°\text{F}$$

with $S = 0.65$, the minimum film thickness from Fig. 12–16 is

$$h_0 = \frac{h_0}{c}c = 0.79(0.001) = 0.000\,79 \text{ in}$$

The coefficient of friction from Fig. 12–18 is

$$f = \frac{fr}{c}\frac{c}{r} = 12.8\frac{0.001}{1} = 0.012\,8$$

The parasitic friction torque T is

$$T = fWr = 0.012\,8(100)(1) = 1.28 \text{ lbf} \cdot \text{in}$$

12–10 Clearance

In designing a journal bearing for thick-film lubrication, the engineer must select the grade of oil to be used, together with suitable values for P, N, r, c, and l. A poor selection of these or inadequate control of them during manufacture or in use may result in a film that is too thin, so that the oil flow is insufficient, causing the bearing to overheat and, eventually, fail. Furthermore, the radial clearance c is difficult to hold accurate during manufacture, and it may increase because of wear. What is the effect of an entire range of radial clearances, expected in manufacture, and what will happen to the bearing performance if c increases because of wear? Most of these questions can be answered and the design optimized by plotting curves of the performance as functions of the quantities over which the designer has control.

Figure 12–25 shows the results obtained when the performance of a particular bearing is calculated for a whole range of radial clearances and is plotted with clearance as the independent variable. The bearing used for this graph is the one of Examples 12–1 to 12–4 with SAE 20 oil at an inlet temperature of 100°F. The graph shows that if the clearance is too tight, the temperature will be too high and the minimum film thickness too low. High temperatures may cause the bearing to fail by fatigue. If the oil film is too thin, dirt particles may be unable to pass without scoring or may embed themselves in the bearing. In either event, there will be excessive wear and friction, resulting in high temperatures and possible seizing.

To investigate the problem in more detail, Table 12–3 was prepared using the two types of preferred running fits that seem to be most useful for journal-bearing design

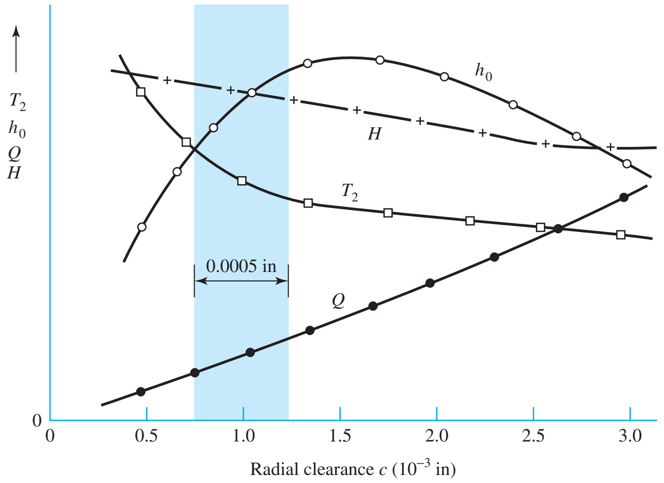


Figure 12–25

A plot of some performance characteristics of the bearing of Exs. 12–1 to 12–4 for radial clearances of 0.0005 to 0.003 in. The bearing outlet temperature is designated T_2. New bearings should be designed for the shaded zone, because wear will move the operating point to the right.

Table 12–3

Maximum, Minimum, and Average Clearances for 1.5-in-Diameter Journal Bearings Based on Type of Fit

Type of Fit	Symbol	Clearance c, in Maximum	Average	Minimum
Close-running	H8/f7	0.001 75	0.001 125	0.000 5
Free-running	H9/d9	0.003 95	0.002 75	0.001 55

Table 12–4

Performance of 1.5-in-Diameter Journal Bearing with Various Clearances. (SAE 20 Lubricant, $T_1 = 100$°F, $N = 30$ r/s, $W = 500$ lbf, $L = 1.5$ in)

c, in	T_2, °F	h_0, in	f	Q, in^3/s	H, Btu/s
0.000 5	226	0.000 38	0.011 3	0.061	0.086
0.001 125	142	0.000 65	0.009 0	0.153	0.068
0.001 55	133	0.000 77	0.008 7	0.218	0.066
0.001 75	128	0.000 76	0.008 4	0.252	0.064
0.002 75	118	0.000 73	0.007 9	0.419	0.060
0.003 95	113	0.000 69	0.007 7	0.617	0.059

(see Table 7–9), p. 389. The results shown in Table 12–3 were obtained by using Eqs. (7–36) and (7–37) of Sec. 7–8. Notice that there is a slight overlap, but the range of clearances for the free-running fit is about twice that of the close-running fit.

The six clearances of Table 12–3 were used in a computer program to obtain the numerical results shown in Table 12–4. These conform to the results of Fig. 12–25, too. Both the table and the figure show that a tight clearance results in a high temperature. Figure 12–26 can be used to estimate an upper temperature limit when the characteristics of the application are known.

It would seem that a large clearance will permit the dirt particles to pass through and also will permit a large flow of oil, as indicated in Table 12–4. This lowers the temperature and increases the life of the bearing. However, if the clearance becomes

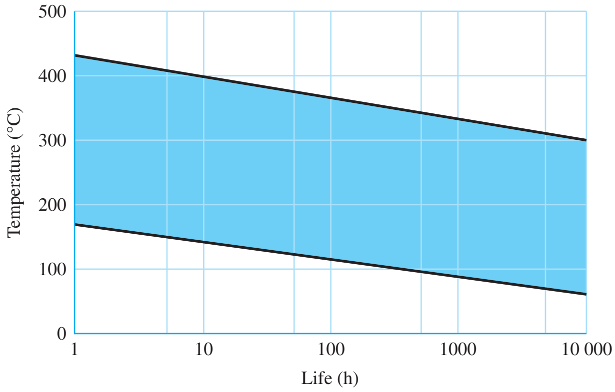


Figure 12–26

Temperature limits for mineral oils. The lower limit is for oils containing antioxidants and applies when oxygen supply is unlimited. The upper limit applies when insignificant oxygen is present. The life in the shaded zone depends on the amount of oxygen and catalysts present. *(Source: M. J. Neale (ed.), Tribology Handbook, Section B1, Newnes-Butterworth, London, 1975.)*

too large, the bearing becomes noisy and the minimum film thickness begins to decrease again.

In between these two limitations there exists a rather large range of clearances that will result in satisfactory bearing performance.

When both the production tolerance and the future wear on the bearing are considered, it is seen, from Fig. 12–25, that the best compromise is a clearance range slightly to the left of the top of the minimum-film-thickness curve. In this way, future wear will move the operating point to the right and increase the film thickness and decrease the operating temperature.

12–11 Pressure-Fed Bearings

The load-carrying capacity of self-contained natural-circulating journal bearings is quite restricted. The factor limiting better performance is the heat-dissipation capability of the bearing. A first thought of a way to increase heat dissipation is to cool the sump with an external fluid such as water. The high-temperature problem is in the film where the heat is generated but cooling is not possible in the film until later. This does not protect against exceeding the maximum allowable temperature of the lubricant. A second alternative is to reduce the *temperature rise* in the film by dramatically increasing the rate of lubricant flow. The lubricant itself is reducing the temperature rise. A water-cooled sump may still be in the picture. To increase lubricant flow, an external pump must be used with lubricant supplied at pressures of tens of pounds per square inch gage. Because the lubricant is supplied to the bearing under pressure, such bearings are called *pressure-fed bearings.*

To force a greater flow through the bearing and thus obtain an increased cooling effect, a common practice is to use a circumferential groove at the center of the bearing, with an oil-supply hole located opposite the load-bearing zone. Such a bearing is shown in Fig. 12–27. The effect of the groove is to create two half-bearings, each having a smaller l/d ratio than the original. The groove divides the pressure-distribution curve into two lobes and reduces the minimum film thickness, but it has wide acceptance among lubrication engineers because such bearings carry more load without overheating.

To set up a method of solution for oil flow, we shall assume a groove ample enough that the pressure drop in the groove itself is small. Initially we will neglect eccentricity and then apply a correction factor for this condition. The oil flow, then, is the amount that flows out of the two halves of the bearing in the direction of the concentric shaft. If we neglect the rotation of the shaft, the flow of the lubricant is

Figure 12–27

Centrally located full annular groove. *(Courtesy of the Cleveland Graphite Bronze Company, Division of Clevite Corporation.)*

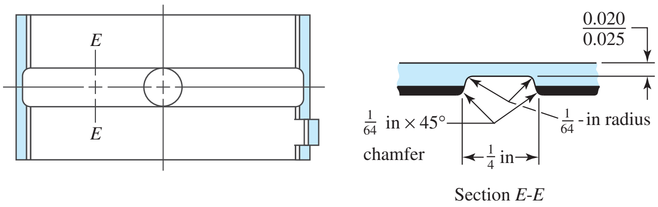
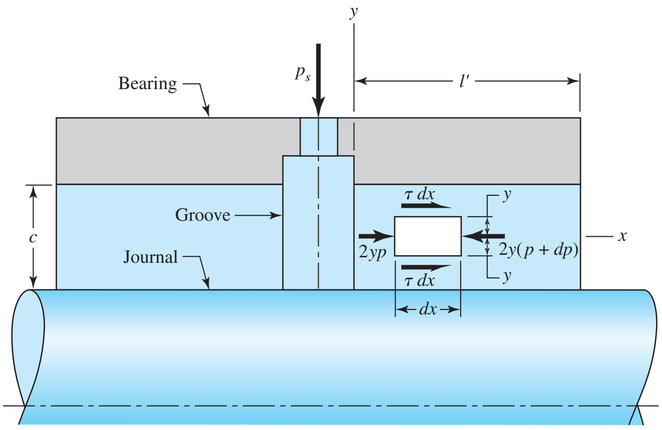


Figure 12–28

Flow of lubricant from a pressure-fed bearing having a central annular groove.



caused by the supply pressure p_s, shown in Fig. 12–28. Laminar flow is assumed, with the pressure varying linearly from $p = p_s$ at $x = 0$, to $p = 0$ at $x = l'$. Consider the static equilibrium of an element of thickness dx, height $2y$, and unit depth. Note particularly that the origin of the reference system has been chosen at the midpoint of the clearance space and symmetry about the x axis is implied with the shear stresses τ being equal on the top and bottom surfaces. The equilibrium equation in the x direction is

$$-2y(p + dp) + 2yp + 2\tau\, dx = 0 \tag{a}$$

Expanding and canceling terms, we find that

$$\tau = y\frac{dp}{dx} \tag{b}$$

Newton's equation for viscous flow [Eq. (12–1)] is

$$\tau = \mu\frac{du}{dy} \tag{c}$$

Now eliminating τ from Eqs. (*b*) and (*c*) gives

$$\frac{du}{dy} = \frac{1}{\mu}\frac{dp}{dx}y \tag{d}$$

Treating dp/dx as a constant and integrating with respect to y gives

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + C_1 \tag{e}$$

At the boundaries, where $y = \pm c/2$, the velocity u is zero. Using one of these conditions in Eq. (e) gives

$$0 = \frac{1}{2\mu}\frac{dp}{dx}\left(\frac{c}{2}\right)^2 + C_1$$

or

$$C_1 = -\frac{c^2}{8\mu}\frac{dp}{dx}$$

Substituting C_1 in Eq. (e) yields

$$u = \frac{1}{8\mu}\frac{dp}{dx}(4y^2 - c^2) \qquad (f)$$

Assuming the pressure varies linearly from p_s to 0 at $x = 0$ to l', respectively, the pressure can be written as

$$p = p_s - \frac{p_s}{l'}x \qquad (g)$$

and therefore the pressure gradient is given by

$$\frac{dp}{dx} = -\frac{p_s}{l'} \qquad (h)$$

We can now substitute Eq. (h) in Eq. (f) and the relationship between the oil velocity and the coordinate y is

$$u = \frac{p_s}{8\mu l'}(c^2 - 4y^2) \qquad (12\text{–}21)$$

Figure 12–29 shows a graph of this relation fitted into the clearance space c so that you can see how the velocity of the lubricant varies from the journal surface to the bearing surface. The distribution is parabolic, as shown, with the maximum velocity occurring at the center, where $y = 0$. The magnitude is, from Eq. (12–21),

$$u_{\max} = \frac{p_s c^2}{8\mu l'} \qquad (i)$$

To consider eccentricity, as shown in Fig. 12–30, the film thickness is $h = c - e\cos\theta$. Substituting h for c in Eq. (i), with the average ordinate of a parabola being two-thirds the maximum, the average velocity at any angular position θ is

$$u_{\text{av}} = \frac{2}{3}\frac{p_s h^2}{8\mu l'} = \frac{p_s}{12\mu l'}(c - e\cos\theta)^2 \qquad (j)$$

We still have a little further to go in this analysis, so please be patient. Now that we have an expression for the lubricant velocity, we can compute the amount of lubricant

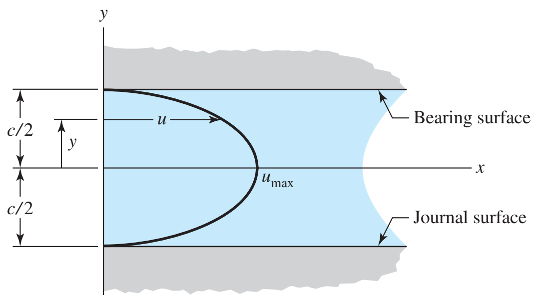
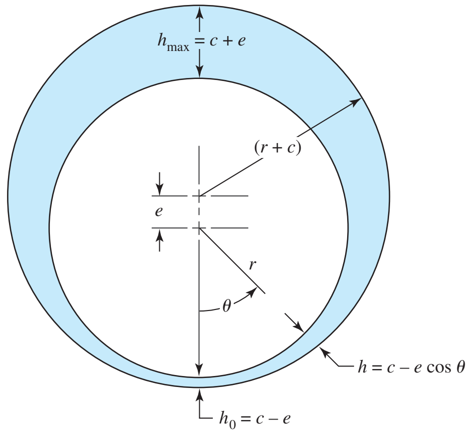


Figure 12–29

Parabolic distribution of the lubricant velocity.

Figure 12–30



that flows out both ends. The elemental side flow at any position θ (Fig. 12–30) is

$$dQ_s = 2u_{av}\,dA = 2u_{av}(rh\,d\theta) \tag{k}$$

where dA is the elemental area. Substituting u_{av} from Eq. (j) and (h) from Fig. 12–30 gives

$$dQ_s = \frac{p_s r}{6\mu l'}(c - e\cos\theta)^3\,d\theta \tag{l}$$

Integrating around the bearing gives the total side flow as

$$Q_s = \int dQ_s = \frac{p_s r}{6\mu l'}\int_0^{2\pi}(c - e\cos\theta)^3\,d\theta = \frac{p_s r}{6\mu l'}(2\pi c^3 + 3\pi\,ce^2)$$

Rearranging, with $\epsilon = e/c$, gives

$$Q_s = \frac{\pi p_s r c^3}{3\mu l'}(1 + 1.5\epsilon^2) \tag{12–22}$$

In analyzing the performance of pressure-fed bearings, the bearing length should be taken as l', as defined in Fig. 12–28. The characteristic pressure in each of the two bearings that constitute the pressure-fed bearing assembly P is given by

$$P = \frac{W/2}{2rl'} = \frac{W}{4rl'} \tag{12–23}$$

The charts for flow variable and flow ratio (Figs. 12–19 and 12–20) do not apply to pressure-fed bearings. Also, the maximum film pressure given by Fig. 12–21 must be increased by the oil supply pressure p_s to obtain the total film pressure.

Since the oil flow has been increased by forced feed, Eq. (12–14) will give a temperature rise that is too high because the side flow carries away all the heat generated. The plumbing in a pressure-fed bearing is depicted schematically in Fig. 12–31. The oil leaves the sump at the externally maintained temperature T_s at the volumetric rate Q_s. The heat gain of the fluid passing through the bearing is

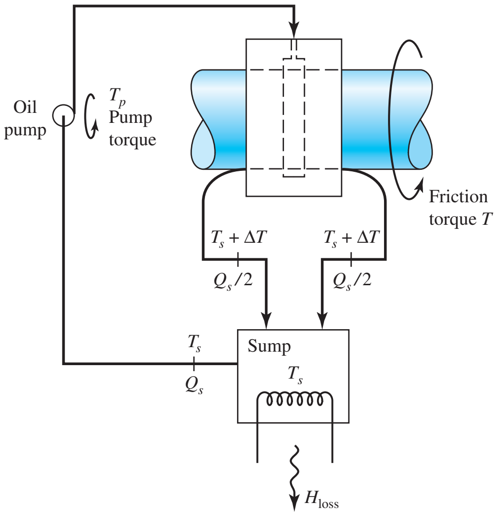
$$H_{gain} = 2\rho C_p(Q_s/2)\Delta T = \rho C_p Q_s \Delta T \tag{m}$$

At steady state, the rate at which the journal does frictional work on the fluid film is

$$H_f = \frac{2\pi TN}{J} = \frac{2\pi f\,WrN}{J} = \frac{2\pi\,WNc}{J}\frac{fr}{c} \tag{n}$$

Figure 12–31

Pressure-fed centrally located full annular-groove journal bearing with external, coiled lubricant sump.



Equating the heat gain to the frictional work and solving for ΔT gives

$$\Delta T = \frac{2\pi WNc}{J\rho C_p Q_s}\frac{fr}{c} \tag{o}$$

Substituting Eq. (12–22) for Q_s in the equation for ΔT gives

$$\Delta T = \frac{2\pi}{J\rho C_p}WNc\frac{fr}{c}\frac{3\mu l'}{(1 + 1.5\epsilon^2)\pi p_s r c^3}$$

The Sommerfeld number may be expressed as

$$S = \left(\frac{r}{c}\right)^2\frac{\mu N}{P} = \left(\frac{r}{c}\right)^2\frac{4rl'\mu N}{W}$$

Solving for $\mu Nl'$ in the Sommerfeld expression; substituting in the ΔT expression; and using $J = 9336$ lbf · in/Btu, $\rho = 0.0311$ lbm/in^3, and $C_p = 0.42$ Btu/(lbm · °F), we find

$$\Delta T_F = \frac{3(fr/c)SW^2}{2J\rho C_p p_s r^4}\frac{1}{(1 + 1.5\epsilon^2)} = \frac{0.0123(fr/c)SW^2}{(1 + 1.5\epsilon^2)p_s r^4} \tag{12–24}$$

where ΔT_F is ΔT in °F. The corresponding equation in SI units uses the bearing load W in kN, lubricant supply pressure p_s in kPa, and the journal radius r in mm:

$$\Delta T_C = \frac{978(10^6)}{1 + 1.5\epsilon^2}\frac{(fr/c)SW^2}{p_s r^4} \tag{12–25}$$

An analysis example of a pressure-fed bearing will be useful.

EXAMPLE 12–6

A circumferential-groove pressure-fed bearing is lubricated with SAE grade 20 oil supplied at a gauge pressure of 30 psi. The journal diameter d_j is 1.750 in, with a unilateral tolerance of −0.002 in. The central circumferential bushing has a diameter d_b of 1.753 in, with a unilateral tolerance of +0.004 in. The l'/d ratio of the two “half-bearings” that constitute the complete pressure-fed bearing is 1/2. The journal angular speed is 3000 rev/min, or 50 rev/s, and the radial steady load is 900 lbf. The external sump is maintained at 120°F as long as the necessary heat transfer does not exceed 800 Btu/h.

(*a*) Find the steady-state average film temperature.

(*b*) Compare h_0, T_{max}, and P_{st} with the Trumpler criteria.

(*c*) Estimate the volumetric side flow Q_s, the heat loss rate H_{loss}, and the parasitic friction torque.

Solution (*a*)

$$r = \frac{d_j}{2} = \frac{1.750}{2} = 0.875 \text{ in}$$

$$c_{min} = \frac{(d_b)_{min} - (d_j)_{max}}{2} = \frac{1.753 - 1.750}{2} = 0.0015 \text{ in}$$

Since $l'/d = 1/2$, $l' = d/2 = r = 0.875$ in. Then the pressure due to the load is

$$P = \frac{W}{4rl'} = \frac{900}{4(0.875)0.875} = 294 \text{ psi}$$

The Sommerfeld number S can be expressed as

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{0.875}{0.0015}\right)^2 \frac{\mu'}{(10^6)} \frac{50}{294} = 0.0579\mu' \qquad (1)$$

We will use a tabulation method to find the average film temperature. The first trial average film temperature $\bar{T}_f$ will be 170°F. Using the Seireg curve fit of Table 12–1, we obtain

$$\mu' = 0.0136 \exp[1271.6/(170 + 95)] = 1.650\ \mu\text{reyn}$$

From Eq. (1)

$$S = 0.0579\mu' = 0.0579(1.650) = 0.0955$$

From Fig. (12–18), $fr/c = 3.3$, and from Fig. (12–16), $\epsilon = 0.80$. From Eq. (12–24),

$$\Delta T_F = \frac{0.0123(3.3)0.0955(900^2)}{[1 + 1.5(0.80)^2]30(0.875^4)} = 91.1°\text{F}$$

$$T_{av} = T_s + \frac{\Delta T}{2} = 120 + \frac{91.1}{2} = 165.6°\text{F}$$

We form a table, adding a second line with $\bar{T}_f = 168.5$°F:

Trial $\bar{T}_f$	μ'	S	fr/c	ϵ	ΔT_F	T_{av}
170	1.65	0.0955	3.3	0.800	91.1	165.6
168.5	1.693	0.0980	3.39	0.792	97.1	168.5

If the iteration had not closed, one could plot trial $\bar{T}_f$ against resulting T_{av} and draw a straight line between them, the intersection with a $\bar{T}_f = T_{av}$ line defining the new trial $\bar{T}_f$.

Answer The result of this tabulation is $\bar{T}_f = 168.5°\text{F}$, $\Delta T_F = 97.1°\text{F}$, and $T_{\max} = 120 + 97.1 = 217.1°\text{F}$

(*b*) Since $h_0 = (1 - \epsilon)c$,

$$h_0 = (1 - 0.792)0.0015 = 0.000\,312 \text{ in}$$

The required four Trumpler criteria, from "Significant Angular Speed" in Sec. 12–7 are

$$h_0 \geq 0.0002 + 0.000\,04(1.750) = 0.000\,270 \text{ in} \qquad \text{(OK)}$$

Answer
$$T_{\max} = T_s + \Delta T = 120 + 97.1 = 217.1°\text{F} \qquad (\leq 250°\text{F. OK})$$

$$P_{st} = \frac{W_{st}}{4rl'} = \frac{900}{4(0.875)0.875} = 294 \text{ psi} \qquad (\leq 300 \text{ psi. OK})$$

Since we are close to the limit on P_{st}, the factor of safety on the load is approximately unity. ($n_d < 2$. Not OK.)

(*c*) From Eq. (12–22),

Answer
$$Q_s = \frac{\pi(30)0.875(0.0015)^3}{3(1.693)10^{-6}(0.875)}[1 + 1.5(0.80)^2] = 0.123 \text{ in}^3\text{/s}$$

$$H_{\text{loss}} = \rho C_p Q_s \Delta T = 0.0311(0.42)0.123(97.1) = 0.156 \text{ Btu/s}$$

or 562 Btu/h or 0.221 hp. The parasitic friction torque T is

Answer
$$T = fWr = \frac{fr}{c}Wc = 3.39(900)0.0015 = 4.58 \text{ lbf} \cdot \text{in}$$

12–12 Loads and Materials

Some help in choosing unit loads and bearing materials is afforded by Tables 12–5 and 12–6. Since the diameter and length of a bearing depend upon the unit load, these tables will help the designer to establish a starting point in the design.

Table 12–5

Range of Unit Loads in Current Use for Sleeve Bearings

	Unit Load	
Application	**psi**	**MPa**
Diesel engines:		
Main bearings	900–1700	6–12
Crankpin	1150–2300	8–15
Wristpin	2000–2300	14–15
Electric motors	120–250	0.8–1.5
Steam turbines	120–250	0.8–1.5
Gear reducers	120–250	0.8–1.5
Automotive engines:		
Main bearings	600–750	4–5
Crankpin	1700–2300	10–15
Air compressors:		
Main bearings	140–280	1–2
Crankpin	280–500	2–4
Centrifugal pumps	100–180	0.6–1.2

The length-diameter ratio l/d of a bearing depends upon whether it is expected to run under thin-film-lubrication conditions. A long bearing (large l/d ratio) reduces the coefficient of friction and the side flow of oil and therefore is desirable where thin-film or boundary-value lubrication is present. On the other hand, where forced-feed or positive lubrication is present, the l/d ratio should be relatively small. The short bearing length results in a greater flow of oil out of the ends, thus keeping the bearing cooler. Current practice is to use an l/d ratio of about unity, in general, and then to increase this ratio if thin-film lubrication is likely to occur and to decrease it for thick-film lubrication or high temperatures. If shaft deflection is likely to be severe, a short bearing should be used to prevent metal-to-metal contact at the ends of the bearings.

You should always consider the use of a partial bearing if high temperatures are a problem, because relieving the non-load-bearing area of a bearing can very substantially reduce the heat generated.

The two conflicting requirements of a good bearing material are that it must have a satisfactory compressive and fatigue strength to resist the externally applied loads and that it must be soft and have a low melting point and a low modulus of elasticity. The second set of requirements is necessary to permit the material to wear or break in, since the material can then conform to slight irregularities and absorb and release foreign particles. The resistance to wear and the coefficient of friction are also important because all bearings must operate, at least for part of the time, with thin-film or boundary lubrication.

Additional considerations in the selection of a good bearing material are its ability to resist corrosion and, of course, the cost of producing the bearing. Some of the commonly used materials are listed in Table 12–6, together with their composition and characteristics.

Bearing life can be increased very substantially by depositing a layer of babbitt, or other white metal, in thicknesses from 0.001 to 0.014 in over steel backup material. In fact, a copper-lead layer on steel to provide strength, combined with a babbitt overlay to enhance surface conformability and corrosion resistance, makes an excellent bearing.

Small bushings and thrust collars are often expected to run with thin-film or boundary lubrication. When this is the case, improvements over a solid bearing material can

Table 12–6

Some Characteristics of Bearing Alloys

Alloy Name	Thickness, in	SAE Number	Clearance Ratio r/c	Load Capacity	Corrosion Resistance
Tin-base babbitt	0.022	12	600–1000	1.0	Excellent
Lead-base babbitt	0.022	15	600–1000	1.2	Very good
Tin-base babbitt	0.004	12	600–1000	1.5	Excellent
Lead-base babbitt	0.004	15	600–1000	1.5	Very good
Leaded bronze	Solid	792	500–1000	3.3	Very good
Copper-lead	0.022	480	500–1000	1.9	Good
Aluminum alloy	Solid		400–500	3.0	Excellent
Silver plus overlay	0.013	17P	600–1000	4.1	Excellent
Cadmium (1.5% Ni)	0.022	18	400–500	1.3	Good
Trimetal 88*				4.1	Excellent
Trimetal 77†				4.1	Very good

*This is a 0.008-in layer of copper-lead on a steel back plus 0.001 in of tin-base babbitt.

†This is a 0.013-in layer of copper-lead on a steel back plus 0.001 in of lead-base babbitt.

be made to add significantly to the life. A powder-metallurgy bushing is porous and permits the oil to penetrate into the bushing material. Sometimes such a bushing may be enclosed by oil-soaked material to provide additional storage space. Bearings are frequently ball-indented to provide small basins for the storage of lubricant while the journal is at rest. This supplies some lubrication during starting. Another method of reducing friction is to indent the bearing wall and to fill the indentations with graphite.

With all these tentative decisions made, a lubricant can be selected and the hydrodynamic analysis made as already presented. The values of the various performance parameters, if plotted as in Fig. 12–25, for example, will then indicate whether a satisfactory design has been achieved or additional iterations are necessary.

12–13 Bearing Types

A bearing may be as simple as a hole machined into a cast-iron machine member. It may still be simple yet require detailed design procedures, as, for example, the two-piece grooved pressure-fed connecting-rod bearing in an automotive engine. Or it may be as elaborate as the large water-cooled, ring-oiled bearings with built-in reservoirs used on heavy machinery.

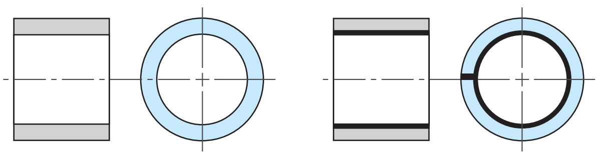
Figure 12–32 shows two types of bushings. The solid bushing is made by casting, by drawing and machining, or by using a powder-metallurgy process. The lined bushing is usually a split type. In one method of manufacture the molten lining material is cast continuously on thin strip steel. The babbitted strip is then processed through presses, shavers, and broaches, resulting in a lined bushing. Any type of grooving may be cut into the bushings. Bushings are assembled as a press fit and finished by boring, reaming, or burnishing.

Flanged and straight two-piece bearings are shown in Fig. 12–33. These are available in many sizes in both thick- and thin-wall types, with or without lining material. A locking lug positions the bearing and effectively prevents axial or rotational movement of the bearing in the housing.

Some typical groove patterns are shown in Fig. 12–34. In general, the lubricant may be brought in from the end of the bushing, through the shaft, or through the bushing. The flow may be intermittent or continuous. The preferred practice is to bring

Figure 12–32

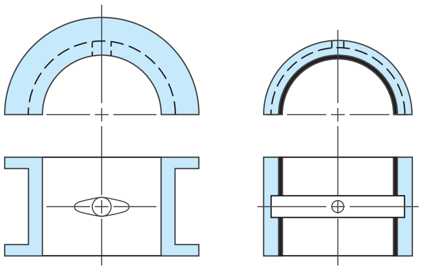
Sleeve bushings.



(*a*) Solid bushing (*b*) Lined bushing

Figure 12–33

Two-piece bushings.



(*a*) Flanged (*b*) Straight

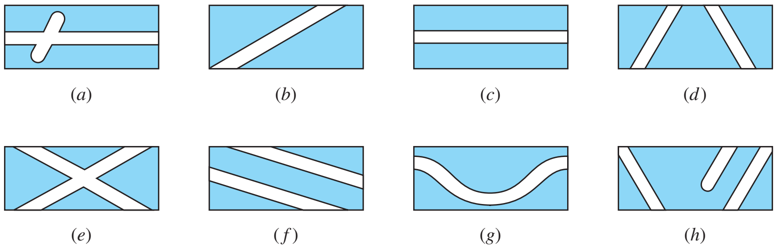


Figure 12–34

Developed views of typical groove patterns. *(Courtesy of the Cleveland Graphite Bronze Company, Division of Clevite Corporation.)*

the oil in at the center of the bushing so that it will flow out both ends, thus increasing the flow and cooling action.

12–14 Thrust Bearings

This chapter is devoted to the study of the mechanics of lubrication and its application to the design and analysis of journal bearings. The design and analysis of thrust bearings is an important application of lubrication theory, too. A detailed study of thrust bearings is not included here, because it would not contribute anything significantly different and because of space limitations. Having studied this chapter, you should experience no difficulty in reading the literature on thrust bearings and applying that knowledge to actual design situations.[11]

Figure 12–35 shows a fixed-pad thrust bearing consisting essentially of a runner sliding over a fixed pad. The lubricant is brought into the radial grooves and pumped into the wedge-shaped space by the motion of the runner. Full-film, or hydrodynamic, lubrication is obtained if the speed of the runner is continuous and sufficiently high, if the lubricant has the correct viscosity, and if it is supplied in sufficient quantity. Figure 12–36 provides a picture of the pressure distribution under conditions of full-film lubrication.

We should note that bearings are frequently made with a flange, as shown in Fig. 12–37. The flange positions the bearing in the housing and also takes a thrust load. Even when it is grooved, however, and has adequate lubrication, such an arrangement is not theoretically a hydrodynamically lubricated thrust bearing. The reason for this is that the clearance space is not wedge-shaped but has a uniform thickness. Similar reasoning would apply to various designs of thrust washers.

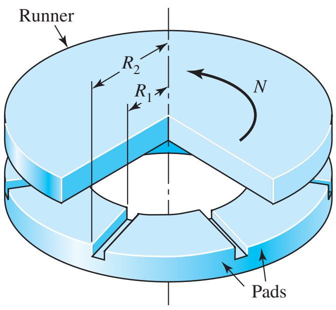


Figure 12–35

Fixed-pad thrust bearing. *(Courtesy of Westinghouse Electric Corporation.)*

[11]Harry C. Rippel, *Cast Bronze Thrust Bearing Design Manual,* International Copper Research Association, Inc., 825 Third Ave., New York, NY 10022, 1967. CBBI, 14600 Detroit Ave., Cleveland, OH, 44107, 1967.

Figure 12–36

Pressure distribution of lubricant in a thrust bearing. *(Courtesy of Copper Research Corporation.)*

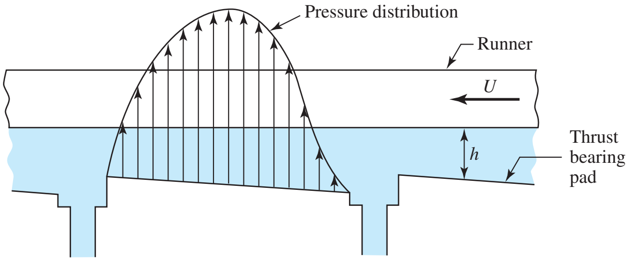
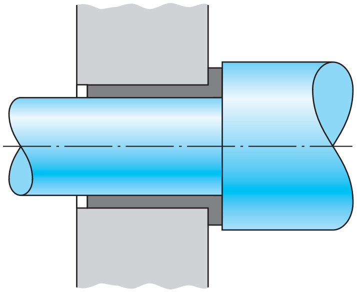


Figure 12–37

Flanged sleeve bearing takes both radial and thrust loads.



12–15 Boundary-Lubricated Bearings

When two surfaces slide relative to each other with only a partial lubricant film between them, *boundary lubrication* is said to exist. Boundary- or thin-film lubrication occurs in hydrodynamically lubricated bearings when they are starting or stopping, when the load increases, when the supply of lubricant decreases, or whenever other operating changes happen to occur. There are, of course, a very large number of cases in design in which boundary-lubricated bearings must be used because of the type of application or the competitive situation.

The coefficient of friction for boundary-lubricated surfaces may be greatly decreased by the use of animal or vegetable oils mixed with the mineral oil or grease. Fatty acids, such as stearic acid, palmitic acid, or oleic acid, or several of these, which occur in animal and vegetable fats, are called *oiliness agents*. These acids appear to reduce friction, either because of their strong affinity for certain metallic surfaces or because they form a soap film that binds itself to the metallic surfaces by a chemical reaction. Thus the fatty-acid molecules bind themselves to the journal and bearing surfaces with such great strength that the metallic asperities of the rubbing metals do not weld or shear.

Fatty acids will break down at temperatures of 250°F or more, causing increased friction and wear in thin-film-lubricated bearings. In such cases the *extreme-pressure,* or EP, lubricants may be mixed with the fatty-acid lubricant. These are composed of chemicals such as chlorinated esters or tricresyl phosphate, which form an organic film between the rubbing surfaces. Though the EP lubricants make it possible to operate at higher temperatures, there is the added possibility of excessive chemical corrosion of the sliding surfaces.

When a bearing operates partly under hydrodynamic conditions and partly under dry or thin-film conditions, a *mixed-film lubrication* exists. If the lubricant is supplied by hand oiling, by drop or mechanical feed, or by wick feed, for example, the bearing is operating under mixed-film conditions. In addition to occurring with a scarcity of lubricant, mixed-film conditions may be present when

- The viscosity is too low.
- The bearing speed is too low.
- The bearing is overloaded.
- The clearance is too tight.
- Journal and bearing are not properly aligned.

Relative motion between surfaces in contact in the presence of a lubricant is called *boundary lubrication*. This condition is present in hydrodynamic film bearings during starting, stopping, overloading, or lubricant deficiency. Some bearings are boundary lubricated (or dry) at all times. To signal this an adjective is placed before the word "bearing." Commonly applied adjectives (to name a few) are thin-film, boundary friction, Oilite, Oiles, and bushed-pin. The applications include situations in which thick film will not develop and there are low journal speed, oscillating journal, padded slides, light loads, and lifetime lubrication. The characteristics include considerable friction, ability to tolerate expected wear without losing function, and light loading. Such bearings are limited by lubricant temperature, speed, pressure, galling, and cumulative wear. Table 12–7 gives some properties of a range of bushing materials.

Linear Sliding Wear

Consider the sliding block depicted in Fig. 12–38, moving along a plate with contact pressure P' acting over area A, in the presence of a coefficient of sliding friction f_s. The linear measure of wear w is expressed in inches or millimeters. The work done by force $f_s PA$ during displacement S is $f_s PAS$ or $f_s PAVt$, where V is the sliding velocity and t is time. The material volume removed due to wear is wA and is proportional to the work done, that is, $wA \propto f_s PAVt$, or

$$wA = KPAVt$$

Table 12–7

Some Materials for Boundary-Lubricated Bearings and Their Operating Limits

Material	Maximum Load, psi	Maximum Temperature, °F	Maximum Speed, fpm	Maximum PV Value*
Cast bronze	4 500	325	1 500	50 000
Porous bronze	4 500	150	1 500	50 000
Porous iron	8 000	150	800	50 000
Phenolics	6 000	200	2 500	15 000
Nylon	1 000	200	1 000	3 000
Teflon	500	500	100	1 000
Reinforced Teflon	2 500	500	1 000	10 000
Teflon fabric	60 000	500	50	25 000
Delrin	1 000	180	1 000	3 000
Carbon-graphite	600	750	2 500	15 000
Rubber	50	150	4 000	
Wood	2 000	150	2 000	15 000

*P = load, psi; V = speed, fpm.

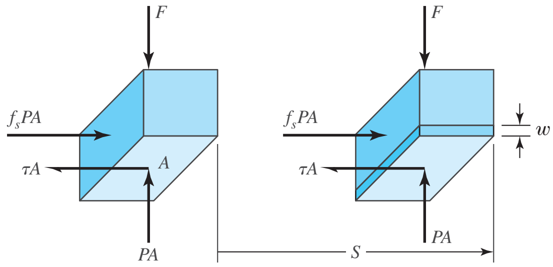


Figure 12–38

Sliding block subjected to wear.

Table 12–8

Wear Factors in U.S. Customary Units*

Source: Oiles America Corp., Plymouth, MI 48170.

Bushing Material	Wear Factor K	Limiting PV
Oiles 800	$3(10^{-10})$	18 000
Oiles 500	$0.6(10^{-10})$	46 700
Polyactal copolymer	$50(10^{-10})$	5 000
Polyactal homopolymer	$60(10^{-10})$	3 000
66 nylon	$200(10^{-10})$	2 000
66 nylon + 15% PTFE	$13(10^{-10})$	7 000
+ 15% PTFE + 30% glass	$16(10^{-10})$	10 000
+ 2.5% MoS_2	$200(10^{-10})$	2 000
6 nylon	$200(10^{-10})$	2 000
Polycarbonate + 15% PTFE	$75(10^{-10})$	7 000
Sintered bronze	$102(10^{-10})$	8 500
Phenol + 25% glass fiber	$8(10^{-10})$	11 500

*dim[K] = $in^3 \cdot min/(lbf \cdot ft \cdot h)$, dim [PV] = psi · ft/min.

Table 12–9

Coefficients of Friction

Source: Oiles America Corp., Plymouth, MI 48170.

Type	Bearing	f_s
Placetic	Oiles 80	0.05
Composite	Drymet ST	0.03
	Toughmet	0.05
Met	Cermet M	0.05
	Oiles 2000	0.03
	Oiles 300	0.03
	Oiles 500SP	0.03

where K is the proportionality factor, which includes f_s, and is determined from laboratory testing. The linear wear is then expressed as

$$w = KPVt \tag{12–26}$$

In US customary units, P is expressed in psi, V in fpm (i.e., ft/min), and t in hours. This makes the units of K $in^3 \cdot min/(lbf \cdot ft \cdot h)$. SI units commonly used for K are $cm^3 \cdot min/(kgf \cdot m \cdot h)$, where 1 kgf = 9.806 N. Tables 12–8 and 12–9 give some wear factors and coefficients of friction from one manufacturer.

Table 12–10

Motion-Related Factor f_1

Mode of Motion	Characteristic Pressure *P*, psi		Velocity *V*, ft/min	f_1*
Rotary	720 or less		3.3 or less	1.0
			3.3–33	1.0–1.3
			33–100	1.3–1.8
	720–3600		3.3 or less	1.5
			3.3–33	1.5–2.0
			33–100	2.0–2.7
Oscillatory	720 or less	>30°	3.3 or less	1.3
			3.3–100	1.3–2.4
		<30°	3.3 or less	2.0
			3.3–100	2.0–3.6
	720–3600	>30°	3.3 or less	2.0
			3.3–100	2.0–3.2
		<30°	3.3 or less	3.0
			3.3–100	3.0–4.8
Reciprocating	720 or less		33 or less	1.5
			33–100	1.5–3.8
	720–3600		33 or less	2.0
			33–100	2.0–7.5

*Values of f_1 based on results over an extended period of time on automotive manufacturing machinery.

Table 12–11

Environmental Factor f_2

Source: Oiles America Corp., Plymouth, MI 48170.

Ambient Temperature, °F	Foreign Matter	f_2
140 or lower	No	1.0
140 or lower	Yes	3.0–6.0
140–210	No	3.0–6.0
140–210	Yes	6.0–12.0

It is useful to include a modifying factor f_1 depending on motion type, load, and speed and an environment factor f_2 to account for temperature and cleanliness conditions (see Tables 12–10 and 12–11). These factors account for departures from the laboratory conditions under which K was measured. Equation (12–26) can now be written as

$$w = f_1 f_2 KPVt \tag{12–27}$$

Wear, then, is proportional to PV, material property K, operating conditions f_1 and f_2, and time t.

Bushing Wear

Consider a pin of diameter D, rotating at speed N, in a bushing of length L, and supporting a stationary radial load F. The nominal pressure P is given by

$$P = \frac{F}{DL} \tag{12–28}$$

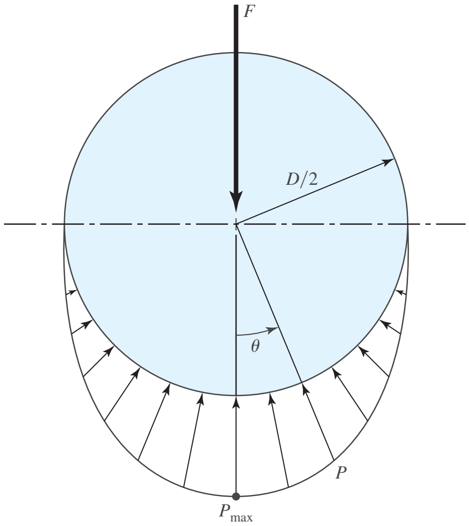


Figure 12–39

Pressure distribution on a boundary-lubricated bushing.

and if N is in rev/min and D is in inches, velocity in ft/min is given by

$$V = \frac{\pi DN}{12} \tag{12–29}$$

Thus PV, in psi · ft/min, is

$$PV = \frac{F}{DL}\frac{\pi DN}{12} = \frac{\pi}{12}\frac{FN}{L} \tag{12–30}$$

Note the independence of PV from the journal diameter D.

A time-wear equation similar to Eq. (12–27) can be written. However, before doing so, it is important to note that Eq. (12–28) provides the nominal value of P. Figure 12–39 provides a more accurate representation of the pressure distribution, which can be written as

$$p = P_{max}\cos\theta \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

The vertical component of $p\,dA$ is $p\,dA\cos\theta = [pL(D/2)\,d\theta]\cos\theta = P_{max}(DL/2)\cos^2\theta\,d\theta$. Integrating this from $\theta = -\pi/2$ to $\pi/2$ yields F. Thus,

$$\int_{-\pi/2}^{\pi/2} P_{max}\left(\frac{DL}{2}\right)\cos^2\theta\,d\theta = \frac{\pi}{4}P_{max}DL = F$$

or

$$P_{max} = \frac{4}{\pi}\frac{F}{DL} \tag{12–31}$$

Substituting V from Eq. (12–29) and P_{max} for P from Eq. (12–31) into Eq. (12–27) gives

$$w = f_1 f_2 K \frac{4}{\pi}\frac{F}{DL}\frac{\pi DNt}{12} = \frac{f_1 f_2 KFNt}{3L} \tag{12–32}$$

In designing a bushing, because of various trade-offs it is recommended that the length/diameter ratio be in the range

$$0.5 \le L/D \le 2 \tag{12–33}$$

EXAMPLE 12–7

An Oiles SP 500 alloy brass bushing is 1 in long with a 1-in bore and operates in a clean environment at 70°F. The allowable wear without loss of function is 0.005 in. The radial load is 700 lbf. The peripheral velocity is 33 ft/min. Estimate the number of revolutions for radial wear to be 0.005 in. See Fig. 12–40 and Table 12–12 from the manufacturer.

Solution

From Table 12–8, $K = 0.6(10^{-10})$ in^3 · min/(lbf · ft · h); Tables 12–10 and 12–11, $f_1 = 1.3$, $f_2 = 1$; and Table 12–12, $PV = 46\,700$ psi · ft/min, $P_{max} = 3560$ psi, $V_{max} = 100$ ft/min. From Eqs. (12–31), (12–29), and (12–30),

$$P_{max} = \frac{4}{\pi}\frac{F}{DL} = \frac{4}{\pi}\frac{700}{(1)(1)} = 891 \text{ psi} < 3560 \text{ psi} \qquad \text{(OK)}$$

$$P = \frac{F}{DL} = \frac{700}{(1)(1)} = 700 \text{ psi}$$

$$V = 33 \text{ ft/min} < 100 \text{ ft/min} \qquad \text{(OK)}$$

$$PV = 700(33) = 23\,100 \text{ psi} \cdot \text{ft/min} < 46\,700 \text{ psi} \cdot \text{ft/min} \qquad \text{(OK)}$$

Equation (12–32) with Eq. (12–29) is

$$w = f_1 f_2 K \frac{4}{\pi}\frac{F}{DL}\frac{\pi DNt}{12} = f_1 f_2 K \frac{4}{\pi}\frac{F}{DL} Vt$$

Figure 12–40

Journal/bushing for Ex. 12–7.

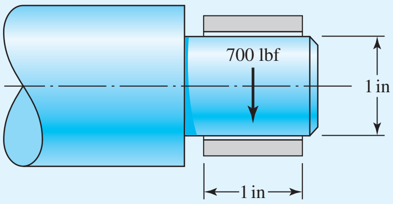


Table 12–12

Oiles 500 SP (SPBN · SPWN) Service Range and Properties

Source: Oiles America Corp., Plymouth, MI 48170.

Service Range	Units	Allowable
Characteristic pressure P_{max}	psi	<3560
Velocity V_{max}	ft/min	<100
PV product	(psi)(ft/min)	<46 700
Temperature *T*	°F	<300
Properties	**Test Method, Units**	**Value**
Tensile strength	(ASTM E8) psi	>110 000
Elongation	(ASTM E8) %	>12
Compressive strength	(ASTM E9) psi	49 770
Brinell hardness	(ASTM E10) HB	>210
Coefficient of thermal expansion	(10^{-5})°C	>1.6
Specific gravity		8.2

Solving for t gives

$$t = \frac{\pi DLw}{4f_1 f_2 KVF} = \frac{\pi(1)(1)0.005}{4(1.3)(1)0.6(10^{-10})33(700)} = 2180 \text{ h} = 130\,770 \text{ min}$$

The rotational speed is

$$N = \frac{12V}{\pi D} = \frac{12(33)}{\pi(1)} = 126 \text{ r/min}$$

Answer $$\text{Cycles} = Nt = 126(130\,770) = 16.5(10^6) \text{ rev}$$

Temperature Rise

At steady state, the rate at which work is done against bearing friction equals the rate at which heat is transferred from the bearing housing to the surroundings by convection and radiation. The rate of heat generation in Btu/h is given by $f_s FV/J$, or

$$H_{\text{gen}} = \frac{f_s F(\pi D)(60N)}{12J} = \frac{5\pi f_s FDN}{J} \tag{12–34}$$

where N is journal speed in rev/min and $J = 778$ ft · lbf/Btu. The rate at which heat is transferred to the surroundings, in Btu/h, is

$$H_{\text{loss}} = \hbar_{\text{CR}} A \Delta T = \hbar_{\text{CR}} A(T_b - T_\infty) = \frac{\hbar_{\text{CR}} A}{2}(T_f - T_\infty) \tag{12–35}$$

where A = housing surface area, ft^2

$\hbar_{\text{CR}}$ = overall combined coefficient of heat transfer, Btu/(h · ft^2 · °F)

T_b = housing metal temperature, °F

T_f = lubricant temperature, °F

The empirical observation that T_b is about midway between T_f and T_∞ has been incorporated in Eq. (12–35). Equating Eqs. (12–34) and (12–35) gives

$$T_f = T_\infty + \frac{10\pi f_s FDN}{J\hbar_{\text{CR}} A} \tag{12–36}$$

Although this equation seems to indicate the temperature rise $T_f - T_\infty$ is independent of length L, the housing surface area generally is a function of L. The housing surface area can be initially estimated, and as tuning of the design proceeds, improved results will converge. If the bushing is to be housed in a pillow block, the surface area can be roughly estimated from

$$A \approx \frac{2\pi DL}{144} \tag{12–37}$$

Substituting Eq. (12–37) into Eq. (12–36) gives

$$T_f \approx T_\infty + \frac{10\pi f_s FDN}{J\hbar_{\text{CR}}(2\pi DL/144)} = T_\infty + \frac{720 f_s FN}{J\hbar_{\text{CR}} L} \tag{12–38}$$

EXAMPLE 12–8 Choose an Oiles 500 bushing to give a maximum wear of 0.001 in for 800 h of use with a 300 rev/min journal and 50 lbf radial load. Use $\hbar_{CR} = 2.7$ Btu/(h · ft^2 · °F), $T_{max} = 300$°F, $f_s = 0.03$, and a design factor $n_d = 2$. Table 12–13 lists the available bushing sizes from the manufacturer.

Solution Using Eq. (12–38) with $n_d F$ for F, $f_s = 0.03$ from Table 12–9, and $\hbar_{CR} = 2.7$ Btu/(h · ft^2 · °F), gives

$$L \geq \frac{720 f_s n_d F N}{J \hbar_{CR}(T_f - T_\infty)} = \frac{720(0.03)2(50)300}{778(2.7)(300 - 70)} = 1.34 \text{ in}$$

From Table 12–13, the smallest available bushing has an ID $= \frac{5}{8}$ in, OD $= \frac{7}{8}$ in, and $L = 1\frac{1}{2}$ in. However, for this case $L/D = 1.5/0.625 = 2.4$, and is outside of the recommendations of Eq. (12–33). Thus, for the first trial, try the bushing with ID $= \frac{3}{4}$ in, OD $= 1\frac{1}{8}$ in, and $L = 1\frac{1}{2}$ in ($L/D = 1.5/0.75 = 2$). Thus,

Table 12–13

Available Bushing Sizes (in inches) of One Manufacturer*

		L													
ID	OD	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	5
$\frac{1}{2}$	$\frac{3}{4}$	•	•	•	•	•									
$\frac{5}{8}$	$\frac{7}{8}$		•	•		•		•							
$\frac{3}{4}$	$1\frac{1}{8}$		•	•		•		•							
$\frac{7}{8}$	$1\frac{1}{4}$			•		•	•	•							
1	$1\frac{3}{8}$			•		•	•	•	•	•					
1	$1\frac{1}{2}$			•		•		•		•					
$1\frac{1}{4}$	$1\frac{5}{8}$					•	•	•	•	•					
$1\frac{1}{2}$	2					•	•	•	•	•					
$1\frac{3}{4}$	$2\frac{1}{4}$						•	•	•	•	•	•	•	•	
2	$2\frac{1}{2}$							•		•	•	•			
$2\frac{1}{4}$	$2\frac{3}{4}$							•		•	•	•			
$2\frac{1}{2}$	3							•		•		•			
$2\frac{3}{4}$	$3\frac{3}{8}$							•		•	•	•			
3	$3\frac{5}{8}$									•	•	•	•		
$3\frac{1}{2}$	$4\frac{1}{8}$									•		•		•	
4	$4\frac{3}{4}$									•		•		•	
$4\frac{1}{2}$	$5\frac{3}{8}$											•		•	•
5	6											•		•	•

*In a display such as this a manufacturer is likely to show catolog numbers where the • appears.

Eq. (12–31): $$P_{\max} = \frac{4}{\pi}\frac{n_d F}{DL} = \frac{4}{\pi}\frac{2(50)}{0.75(1.5)} = 113 \text{ psi} < 3560 \text{ psi} \qquad \text{(OK)}$$

$$P = \frac{n_d F}{DL} = \frac{2(50)}{0.75(1.5)} = 88.9 \text{ psi}$$

Eq. (12–29): $$V = \frac{\pi DN}{12} = \frac{\pi(0.75)300}{12} = 58.9 \text{ ft/min} < 100 \text{ ft/min} \qquad \text{(OK)}$$

$$PV = 88.9(58.9) = 5240 \text{ psi} \cdot \text{ft/min} < 46\,700 \text{ psi} \cdot \text{ft/min} \qquad \text{(OK)}$$

From Table 12–10, interpolation gives

V	f_1	
33	1.3	
58.9	f_1	from which $f_1 = 1.50$
100	1.8	

Eq. (12–32), with Tables 12–8 and 12–10:

$$w = \frac{f_1 f_2 K n_d F N t}{3L} = \frac{1.50(1)6(10^{-11})2(50)300(800)}{3(1.5)} = 0.000480 \text{ in} < 0.001 \text{ in} \quad \text{(OK)}$$

Answer Select ID $= \frac{3}{4}$ in, OD $= 1\frac{1}{8}$ in, and $L = 1\frac{1}{2}$ in.

PROBLEMS

12–1 A full journal bearing has a journal diameter of 25 mm, with a unilateral tolerance of −0.03 mm. The bushing bore has a diameter of 25.03 mm and a unilateral tolerance of 0.04 mm. The l/d ratio is 1/2. The load is 1.2 kN and the journal runs at 1100 rev/min. If the average viscosity is 55 mPa · s, find the minimum film thickness, the power loss, and the side flow for the minimum clearance assembly.

12–2 A full journal bearing has a journal diameter of 32 mm, with a unilateral tolerance of −0.012 mm. The bushing bore has a diameter of 32.05 mm and a unilateral tolerance of 0.032 mm. The bearing is 64 mm long. The journal load is 1.75 kN and it runs at a speed of 900 rev/min. Using an average viscosity of 55 mPa · s find the minimum film thickness, the maximum film pressure, and the total oil-flow rate for the minimum clearance assembly.

12–3 A journal bearing has a journal diameter of 3.000 in, with a unilateral tolerance of −0.001 in. The bushing bore has a diameter of 3.005 in and a unilateral tolerance of 0.004 in. The bushing is 1.5 in long. The journal speed is 600 rev/min and the load is 800 lbf. For both SAE 10 and SAE 40, lubricants, find the minimum film thickness and the maximum film pressure for an operating temperature of 150°F for the minimum clearance assembly.

12–4 A journal bearing has a journal diameter of 3.250 in with a unilateral tolerance of −0.003 in. The bushing bore has a diameter of 3.256 in and a unilateral tolerance of 0.004 in. The bushing is 3 in long and supports a 800-lbf load. The journal speed is 1000 rev/min. Find the minimum oil film thickness and the maximum film pressure for both SAE 20 and SAE 20W-40 lubricants, for the tightest assembly if the operating film temperature is 150°F.

12–5 A full journal bearing has a journal with a diameter of 2.000 in and a unilateral tolerance of −0.0012 in. The bushing has a bore with a diameter of 2.0024 and a unilateral tolerance of 0.002 in. The bushing is 1 in long and supports a load of 600 lbf at a speed of 800 rev/min. Find the minimum film thickness, the power loss, and the total lubricant flow if the average film temperature is 130°F and SAE 20 lubricant is used. The tightest assembly is to be analyzed.

12–6 A full journal bearing has a shaft journal diameter of 25 mm with a unilateral tolerance of −0.01 mm. The bushing bore has a diameter of 25.04 mm with a unilateral tolerance of 0.03 mm. The l/d ratio is unity. The bushing load is 1.25 kN, and the journal rotates at 1200 rev/min. Analyze the minimum clearance assembly if the average viscosity is 50 mPa · s to find the minimum oil film thickness, the power loss, and the percentage of side flow.

12–7 A full journal bearing has a shaft journal with a diameter of 1.25 in and a unilateral tolerance of −0.0006 in. The bushing bore has a diameter of 1.252 in with a unilateral tolerance of 0.0014 in. The bushing bore is 2 in in length. The bearing load is 620 lbf and the journal rotates at 1120 rev/min. Analyze the minimum clearance assembly and find the minimum film thickness, the coefficient of friction, and the total oil flow if the average viscosity is 8.5 μreyn.

12–8 A journal bearing has a shaft diameter of 75.00 mm with a unilateral tolerance of −0.02 mm. The bushing bore has a diameter of 75.10 mm with a unilateral tolerance of 0.06 mm. The bushing is 36 mm long and supports a load of 2 kN. The journal speed is 720 rev/min. For the minimum clearance assembly find the minimum film thickness, the heat loss rate, and the maximum lubricant pressure for SAE 20 and SAE 40 lubricants operating at an average film temperature of 60°C.

12–9 A full journal bearing is 28 mm long. The shaft journal has a diameter of 56 mm with a unilateral tolerance of −0.012 mm. The bushing bore has a diameter of 56.05 mm with a unilateral tolerance of 0.012 mm. The load is 2.4 kN and the journal speed is 900 rev/min. For the minimum clearance assembly find the minimum oil-film thickness, the power loss, and the side flow if the operating temperature is 65°C and SAE 40 lubricating oil is used.

12–10 A $1\frac{1}{4}$- × $1\frac{1}{4}$-in sleeve bearing supports a load of 700 lbf and has a journal speed of 3600 rev/min. An SAE 10 oil is used having an average temperature of 160°F. Using Fig. 12–16, estimate the radial clearance for minimum coefficient of friction f and for maximum load-carrying capacity W. The difference between these two clearances is called the clearance range. Is the resulting range attainable in manufacture?

12–11 A full journal bearing has a shaft diameter of 3.000 in with a unilateral tolerance of −0.0004 in. The l/d ratio is unity. The bushing has a bore diameter of 3.003 in with a unilateral tolerance of 0.0012 in. The SAE 40 oil supply is in an axial-groove sump with a steady-state temperature of 140°F. The radial load is 675 lbf. Estimate the average film temperature, the minimum film thickness, the heat loss rate, and the lubricant side-flow rate for the minimum clearance assembly, if the journal speed is 10 rev/s.

12–12 A $2\frac{1}{2}$ × $2\frac{1}{2}$-in sleeve bearing uses grade 20 lubricant. The axial-groove sump has a steady-state temperature of 110°F. The shaft journal has a diameter of 2.500 in with a unilateral tolerance of −0.001 in. The bushing bore has a diameter of 2.504 in with a unilateral tolerance of 0.001 in. The journal speed is 1120 rev/min and the radial load is 1200 lbf. Estimate

(*a*) The magnitude and location of the minimum oil-film thickness.
(*b*) The eccentricity.
(*c*) The coefficient of friction.
(*d*) The power loss rate.
(*e*) Both the total and side oil-flow rates.
(*f*) The maximum oil-film pressure and its angular location.
(*g*) The terminating position of the oil film.
(*h*) The average temperature of the side flow.
(*i*) The oil temperature at the terminating position of the oil film.

12–13 A set of sleeve bearings has a specification of shaft journal diameter of 1.250 in with a unilateral tolerance of −0.001 in. The bushing bore has a diameter of 1.252 in with a unilateral tolerance of 0.003 in. The bushing is $1\frac{1}{4}$ in long. The radial load is 250 lbf and the shaft rotational speed is 1750 rev/min. The lubricant is SAE 10 oil and the axial-groove sump temperature at steady state T_s is 120°F. For the $c_{\min}$, c_{median}, and $c_{\max}$ assemblies analyze the bearings and observe the changes in S, ϵ, f, Q, Q_s, ΔT, $T_{\max}$, $\bar{T}_f$, and h_0.

12–14 An interpolation equation was given by Raimondi and Boyd, and it is displayed as Eq. (12–16). This equation is a good candidate for a computer program. Write such a program for interactive use. Once ready for service it can save time and reduce errors. Another version of this program can be used with a subprogram that contains curve fits to Raimondi and Boyd charts for computer use.

12–15 A natural-circulation pillow-block bearing with $l/d = 1$ has a journal diameter D of 2.500 in with a unilateral tolerance of −0.001 in. The bushing bore diameter B is 2.504 in with a unilateral tolerance of 0.004 in. The shaft runs at an angular speed of 1120 rev/min; the bearing uses SAE grade 20 oil and carries a steady load of 300 lbf in shaft-stirred air at 70°F with $\alpha = 1$. The lateral area of the pillow-block housing is 60 in^2. Perform a design assessment using minimum radial clearance for a load of 600 lbf and 300 lbf. Use Trumpler's criteria.

12–16 An eight-cylinder diesel engine has a front main bearing with a journal diameter of 3.500 in and a unilateral tolerance of −0.003 in. The bushing bore diameter is 3.505 in with a unilateral tolerance of +0.005 in. The bushing length is 2 in. The pressure-fed bearing has a central annular groove 0.250 in wide. The SAE 30 oil comes from a sump at 120°F using a supply pressure of 50 psig. The sump's heat-dissipation capacity is 5000 Btu/h per bearing. For a minimum radial clearance, a speed of 2000 rev/min, and a radial load of 4600 lbf, find the average film temperature and apply Trumpler's criteria in your design assessment.

12–17 A pressure-fed bearing has a journal diameter of 50.00 mm with a unilateral tolerance of −0.05 mm. The bushing bore diameter is 50.084 mm with a unilateral tolerance of 0.10 mm. The length of the bushing is 55 mm. Its central annular groove is 5 mm wide and is fed by SAE 30 oil is 55°C at 200 kPa supply gauge pressure. The journal speed is 2880 rev/min carrying a load of 10 kN. The sump can dissipate 300 watts per bearing if necessary. For minimum radial clearances, perform a design assessment using Trumpler's criteria.

12–18 Design a central annular-groove pressure-fed bearing with an l'/d ratio of 0.5, using SAE grade 20 oil, the lubricant supplied at 30 psig. The exterior oil cooler can maintain the sump temperature at 120°F for heat dissipation rates up to 1500 Btu/h. The load to be carried is 900 lbf at 3000 rev/min. The groove width is $\frac{1}{4}$ in. Use nominal journal diameter d as one design variable and c as the other. Use Trumpler's criteria for your adequacy assessment.

12–19 Repeat design problem Prob. 12–18 using the nominal bushing bore B as one decision variable and the radial clearance c as the other. Again, Trumpler's criteria to be used.

12–20 Table 12–1 gives the Seireg and Dandage curve fit approximation for the absolute viscosity in customary U.S. engineering units. Show that in SI units of mPa · s and a temperature of C degrees Celsius, the viscosity can be expressed as

$$\mu = 6.89(10^6)\mu_0 \exp[(b/(1.8C + 127))]$$

where μ_0 and b are from Table 12–1. If the viscosity μ_0' is expressed in μreyn, then

$$\mu = 6.89\mu_0' \exp[(b/(1.8C + 127))]$$

What is the viscosity of a grade 50 oil at 70°C? Compare your results with Fig. 12–13.

12–21 For Prob. 12–18 a satisfactory design is

$$d = 2.000^{+0}_{-0.001} \text{ in} \qquad b = 2.005^{+0.003}_{-0} \text{ in}$$

Double the size of the bearing dimensions and quadruple the load to 3600 lbf.
(*a*) Analyze the scaled-up bearing for median assembly.
(*b*) Compare the results of a similar analysis for the 2-in bearing, median assembly.

12–22 An Oiles SP 500 alloy brass bushing is 0.75 in long with a 0.75-in dia bore and operates in a clean environment at 70°F. The allowable wear without loss of function is 0.004 in. The radial load is 400 lbf. The shaft speed is 250 rev/min. Estimate the number of revolutions for radial wear to be 0.004 in.

12–23 Choose an Oiles SP 500 alloy brass bushing to give a maximum wear of 0.002 in for 1000 h of use with a 200 rev/min journal and 100 lbf radial load. Use $\hbar_{CR} = 2.7$ Btu/(h · ft^2 · °F), $T_{max} = 300$°F, $f_s = 0.03$, and a design factor $n_d = 2$. The bearing is to operate in a clean environment at 70°F. Table 12–13 lists the bushing sizes available from the manufacturer.

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